Astropartículas: Mensajeros de lo visible y lo invisible

Juan Barranco Monarca

XII Taller de la División de Gravitación y Física Matemática
Guadalajara, 27 de Noviembre-1ro de diciembre del 2017

Astropartícula → astro + partícula

Astro: del latín *astrum* y este del griego antiguo *astron*: estrella
Real academia de la lengua: Cada uno de los innumerables cuerpos celestes que pueblan el Firmamento.

Partícula: Componente pequeño de la materia

Una astropartícula es una partícula que proviene de algún cuerpo celeste:

Una astropartícula es un mensajero del universo

Astropartículas

- Es una rama de la física de partículas que estudia a las partículas elementales de origen astronómico y su relación con astrofísica y cosmología.
- La física de astropartículas es un nuevo campo que surge de la intersección de:
  - Física de partículas
  - Astronomía
  - Astrofísica
  - Física de detectores
  - Cosmología
  - Física del estado sólido
  - Relatividad
- Debido en parte al descubrimiento de las oscilaciones de los neutrinos, este nuevo campo ha tenido un rápido crecimiento, tanto teórico como experimental a partir de inicios del siglo XXI.

Ingredientes básicos

Producción

Propagación

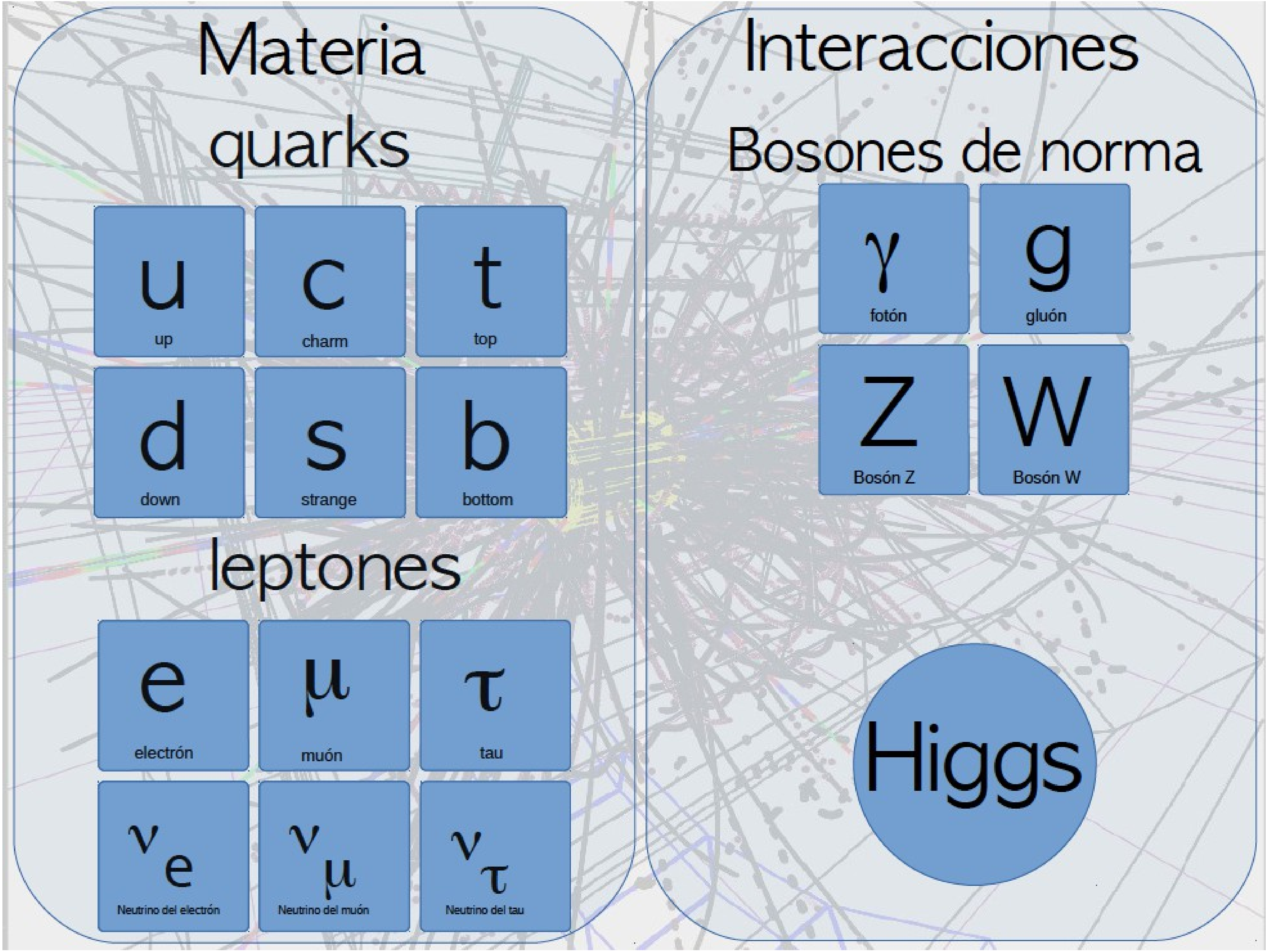
Detección

1. Mecanismo de produccón
   - procesos astrofísicos $\rightarrow$ reacciones nucleares
   - Astrofísica (Estrellas de neutrones, supernova, Remanentes de SN, AGN)
   - decaimientos $\frac{d\Gamma}{dE}$
2. Propagación
   - Interacción de la partícula con su medio $\lambda_{mfp} \sim \frac{1}{\sigma n}$
3. Detección
   - Reacción de detección $\frac{d\sigma}{dE}$
   - Características del detector

Entendimiento de las interacciones de las partículas

Good news!

We do understand the particles: The standard model of elementary particles


Materia
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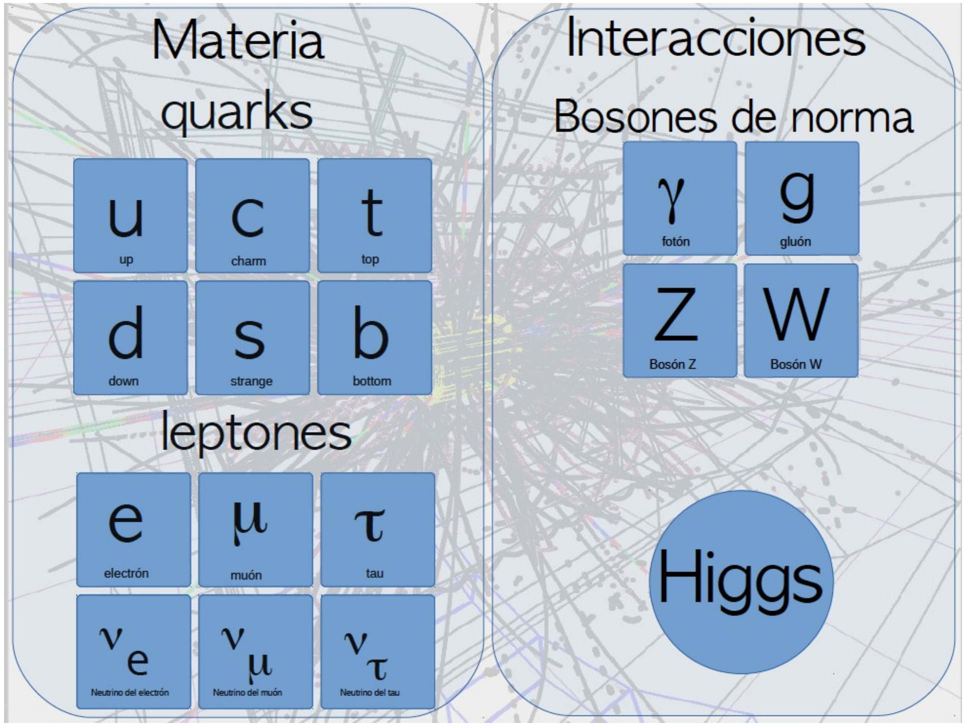
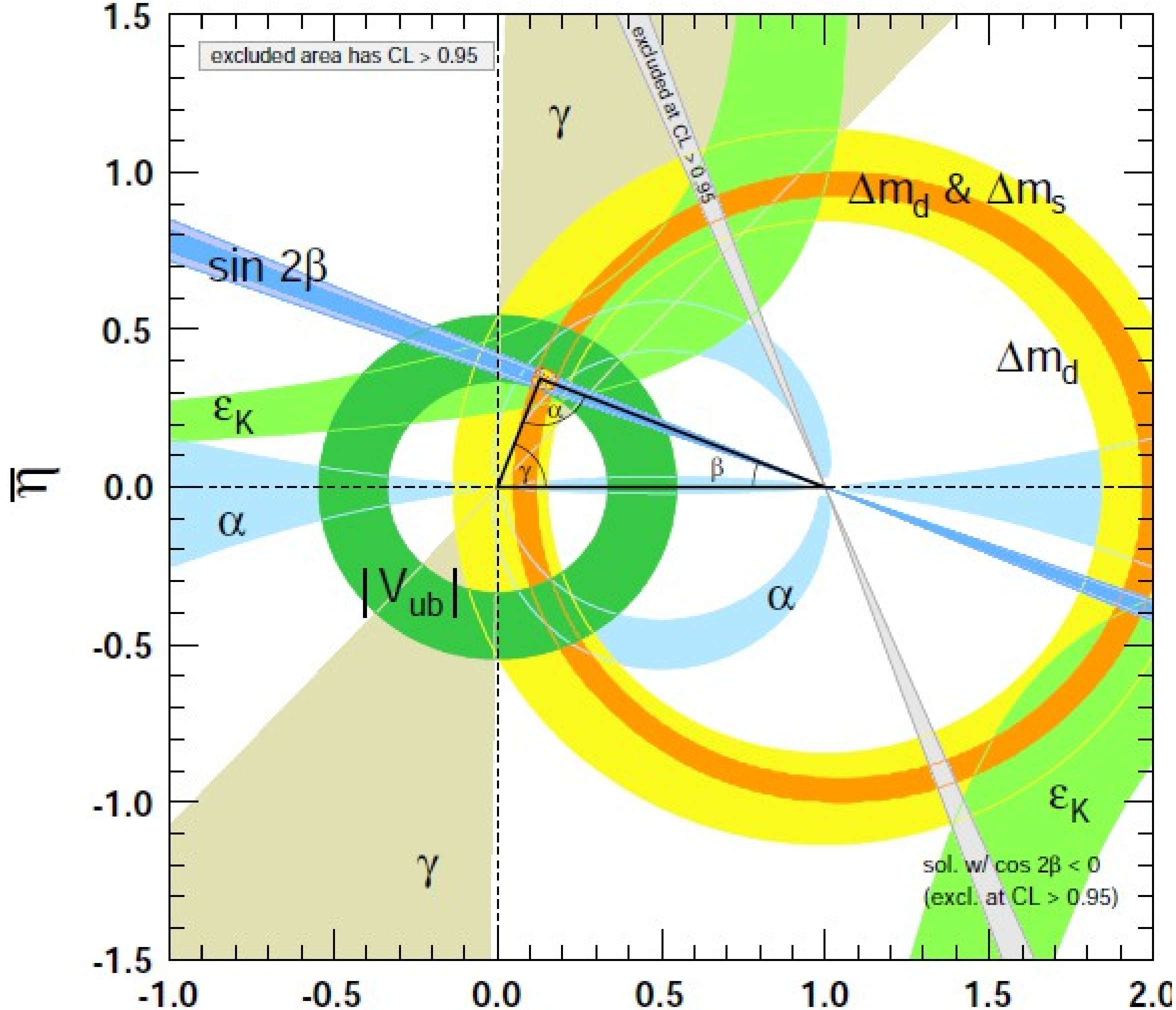
Neutrino del electrón
ν_μ
Neutrino del muón
ν_τ
Neutrino del tau
Interacciones
Bosones de norma
γ
fotón
g
gluón
Z
Bosón Z
W
Bosón W
Higgs

Standard model

A concordance model

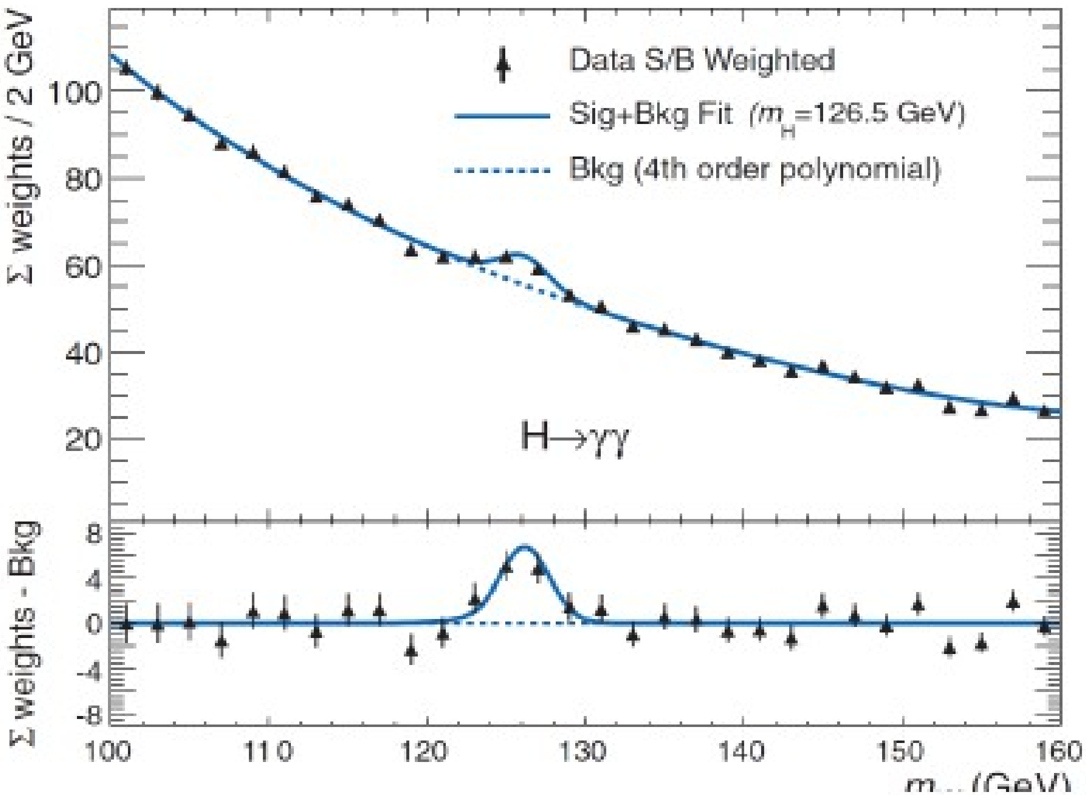
19 free parameters + neutrino parameters

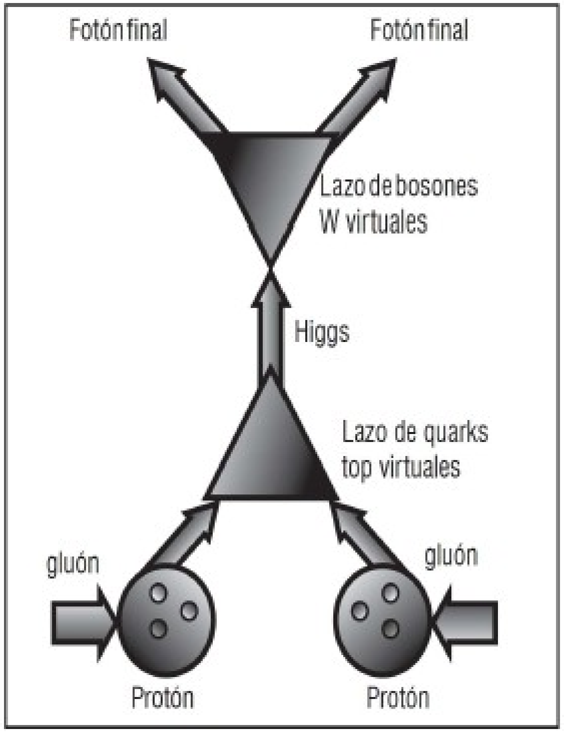




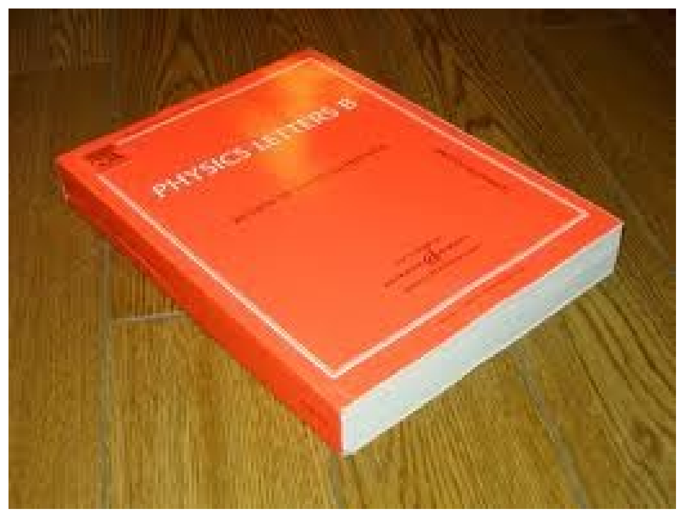
$$V_{\mathrm{CKM}} = \begin{pmatrix} 0{,}97428 \pm 0{,}00015 & 0{,}2253 \pm 0{,}0007 & 0{,}00347^{+0{,}00016}_{-0{,}00012} \\ 0{,}2252 \pm 0{,}0007 & 0{,}97345^{+0{,}00015}_{-0{,}00016} & 0{,}0410^{+0{,}0011}_{-0{,}0007} \\ 0{,}00862^{+0{,}00026}_{-0{,}00020} & 0{,}0403^{+0{,}0011}_{-0{,}0007} & 0{,}999152^{+0{,}000030}_{-0{,}000045} \end{pmatrix}$$

And the Higgs



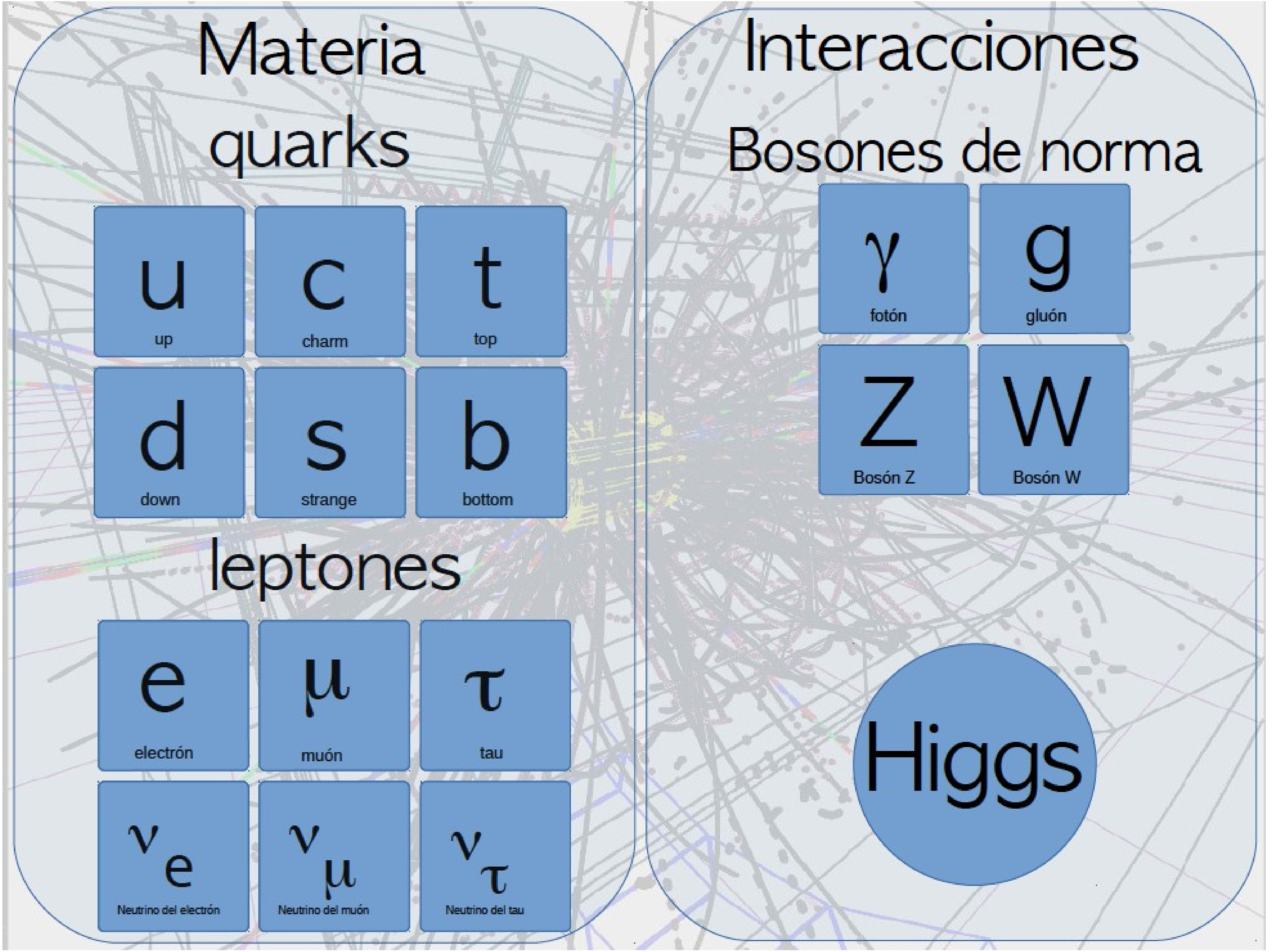


Concordance model: Hundred of experiments that have been verified and they match by fixing only those 19 parameters



Fundamental (elementary) particles

Fundamental interactions


Materia
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Neutrino del electrón
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Neutrino del muón
ντ
Neutrino del tau
Interacciones
Bosones de norma
γ
fotón
g
gluón
Z
Bosón Z
W
Bosón W
Higgs

What is an elementary particle?

Una condición sine qua non para la existencia implica cierta dosis de conservación

Alfonso Reyes. La experiencia Literaria

(A sine qua non condition for the existence requires certain amount of conservation)

Sine qua non : an essential action

Noether theorem

Symmetries ------> Conserved quantities

Lagrangian--------> Quantum numbers

Quark flavor properties[72]

Name	Symbol	Mass (MeV/c^2)*	J	B	Q (e)	I_3	C	S	T	B'	Antiparticle	Antiparticle symbol
First generation												
Up	u	2.3 ±0.7 ± 0.5	$\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{2}{3}$	$+\frac{1}{2}$	0	0	0	0	Antiup	$\bar{u}$
Down	d	4.8 ±0.5 ± 0.3	$\frac{1}{2}$	$+\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	0	0	0	0	Antidown	$\bar{d}$
Second generation												
Charm	c	1275 ± 25	$\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{2}{3}$	0	+1	0	0	0	Anticharm	$\bar{c}$
Strange	s	95 ± 5	$\frac{1}{2}$	$+\frac{1}{3}$	$-\frac{1}{3}$	0	0	−1	0	0	Antistrange	$\bar{s}$
Third generation												
Top	t	173 210 ± 510 ± 710	$\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{2}{3}$	0	0	0	+1	0	Antitop	$\bar{t}$
Bottom	b	4180 ± 30	$\frac{1}{2}$	$+\frac{1}{3}$	$-\frac{1}{3}$	0	0	0	0	−1	Antibottom	$\bar{b}$

J = total angular momentum, B = baryon number, Q = electric charge, I_3 = isospin, C = charm, S = strangeness, T = topness, B' = bottomness

Nature is relativistic and quantum, i.e. QFT

$$
\begin{aligned}
i\hbar \, \frac{\partial \psi}{\partial t} &= H\psi \\
H &= c\vec{\alpha}\,(\vec{p} - \frac{e}{c}\,\vec{A}) + \beta m c^2 + e\varphi\,\mathbf{1}
\end{aligned}
$$

Consider the non-relativistic limit of the Dirac equation:

$$
H = \frac{1}{2m}\,(\vec{p} - e\vec{A})^2 + V - 2\mu_B\,\vec{S}\cdot\vec{B} + \frac{1}{2}\left(\frac{1}{m}\right)^2 \frac{1}{r}\,V'\,\vec{L}\cdot\vec{S}
$$

Correct interaction term.

Home work: what about spin 1 particles?

Interactions: Local gauge invariance (QED)

Start with a Dirac Lagrangian:

$$\mathfrak{L} = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi$$

Is it invariant under the transformation:

$$\bar{\psi} \to e^{-i\alpha(x)}\bar{\psi} \quad ?$$

No: $$\partial_{\mu}\psi \to e^{i\alpha(x)}\partial_{\mu}\psi + ie^{i\alpha(x)}\psi\,\partial_{\mu}\alpha$$

Modify the derivative: $$D_{\mu} \equiv \partial_{\mu} - ieA_{\mu}$$

$$D_{\mu}\psi \to e^{i\alpha(x)}D_{\mu}\psi$$

But also, impose a rule for the potential:

$$A_\mu \rightarrow A_\mu + \frac{1}{e}\partial_\mu \alpha$$

With those assumtions, verify:

$$\begin{aligned}\mathcal{L} &= i\bar{\psi}\gamma^\mu D_\mu \psi - m\bar{\psi}\psi \\ &= \bar{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi + e\bar{\psi}\gamma^\mu \psi A_\mu\end{aligned}$$

Does it look familiar to you?

Finally, add a kinetic term:

$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi + e\bar{\psi}\gamma^\mu A_\mu \psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Interaction fo a fermion with a electromagnetic field, the third term a current.

$$\mathcal{L} = \underbrace{\bar{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi}_{\substack{\text{Kinetic energy} \\ \text{and mass of } \psi}} - \underbrace{e\bar{\psi}\gamma^\mu Q \psi A_\mu}_{\text{Interaction}} - \underbrace{\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\substack{\text{Kinetic energy} \\ \text{of } A_\mu}}.$$

Local gauge invariance (QCD)

Start again with the Lagrangian:

$$\mathcal{L}_0 = \bar{q}_j\left(i\gamma^\mu\partial_\mu - m\right)q_j$$

But now q_1, q_2, q_3 denote three color fields.

Is this Lagrangian invariant under this transformation?

$$q(x) \rightarrow Uq(x) \equiv e^{i\alpha_a(x)T_a}q(x)$$

Here U is a 3x3 matrix, and

$$[T_a, T_b] = if_{abc}T_c$$

As in previous case, if we add 8 gauge fields:

$$G_\mu^a \rightarrow G_\mu^a - \frac{1}{g}\partial_\mu\alpha_a - f_{abc}\alpha_b G_\mu^c$$

And the covariant derivative

$$D_\mu = \partial_\mu + igT_a G_\mu^a$$

Then, Lagrangian is invariant, although, expressing in normal derivatives:

$$\mathcal{L} = \bar{q}\left(i\gamma^\mu \partial_\mu - m\right)q - g(\bar{q}\gamma^\mu T_a q)G_\mu^a$$

Adding a kinetic term for the fields:

$$\mathcal{L} = \bar{q}\left(i\gamma^\mu \partial_\mu - m\right)q - g(\bar{q}\gamma^\mu T_a q)G_\mu^a - \frac{1}{4}G_{\mu\nu}^a G_a^{\mu\nu}$$

This is the QCD Lagrangian.

Electro-Weak interactions:

$$\mathcal{L} = -\tfrac{1}{4}\mathbf{W}_{\mu\nu}\cdot\mathbf{W}^{\mu\nu} - \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

$\left\{\right.$ $W^{\pm}$, Z, γ kinetic energies and self-interactions

$$+\bar{L}\gamma^{\mu}\left(i\partial_{\mu} - g\frac{1}{2}\boldsymbol{\tau}\cdot\mathbf{W}_{\mu} - g'\frac{Y}{2}B_{\mu}\right)L$$
$$+\bar{R}\gamma^{\mu}\left(i\partial_{\mu} - g'\frac{Y}{2}B_{\mu}\right)R$$

$\left\{\right.$ lepton and quark kinetic energies and their interactions with $W^{\pm}$, Z, γ

$$+\left|\left(i\partial_{\mu} - g\frac{1}{2}\boldsymbol{\tau}\cdot\mathbf{W}_{\mu} - g'\frac{Y}{2}B_{\mu}\right)\phi\right|^{2} - V(\phi)$$

$\left\{\right.$ $W^{\pm}$, Z, γ, and Higgs masses and couplings

$$-(G_1\bar{L}\phi R + G_2\bar{L}\phi_c R + \text{hermitian conjugate}).$$

$\left\{\right.$ lepton and quark masses and coupling to Higgs

Where:

$$\mathbf{W}_{\mu\nu} = \partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu - g\mathbf{W}_\mu \times \mathbf{W}_\nu$$

And we have introduced the Higgs potential:

$$V(\phi) = \mu^2\phi^\dagger\phi + \lambda\left(\phi^\dagger\phi\right)^2$$

Now, let us re-write some terms:

$$-g\,\bar{\mathsf{L}}\,\gamma^\mu\left(\frac{\tau^1}{2}W_\mu^1 + \frac{\tau^2}{2}W_\mu^2\right)\mathsf{L} - g\,\bar{\mathsf{L}}\,\gamma^\mu\frac{\tau^3}{2}\mathsf{L}\,W_\mu^3 - \frac{g'}{2}Y\bar{\mathsf{L}}\,\gamma^\mu\mathsf{L}\,B_\mu$$

The first term is *charged* and can be written as

$$\mathcal{L}_{\text{leptons}}^{\mathsf{L}(\pm)} = -\frac{g}{2}\,\bar{\mathsf{L}}\,\gamma^\mu\begin{pmatrix} 0 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & 0 \end{pmatrix}\mathsf{L}\,.$$

This suggests the definition of the *charged gauge bosons* as

$$\boxed{W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp W_\mu^2)}\,,$$

Then, the charged electro weak interaction will be:

$$\mathcal{L}^{\mathsf{L}(\pm)}_{\text{leptons}} = -\frac{g}{2\sqrt{2}} \left[\bar{\nu}\gamma^{\mu}(1-\gamma_5)\ell \, W^{+}_{\mu} + \bar{\ell}\gamma^{\mu}(1-\gamma_5)\nu \, W^{-}_{\mu} \right]$$

Now, consider the neutral term:

$$\begin{aligned} \mathcal{L}^{(\mathsf{L}+\mathsf{R})(0)}_{\text{leptons}} &= -g \, \bar{\mathsf{L}} \left(\gamma^{\mu} \frac{\tau^3}{2} \right) \mathsf{L} \, W^3_{\mu} - \frac{g'}{2} \left(\bar{\mathsf{L}}\gamma^{\mu} Y \mathsf{L} + \bar{\mathsf{R}}\gamma^{\mu} Y \mathsf{R} \right) B_{\mu} \\ &= -g \, J^{\mu}_3 \, W^3_{\mu} - \frac{g'}{2} \, J^{\mu}_Y \, B_{\mu} \, , \end{aligned}$$

where

$$\begin{aligned} J^{\mu}_3 &= \frac{1}{2}(\bar{\nu}_L\gamma^{\mu}\nu_L - \bar{\ell}_L\gamma^{\mu}\ell_L) \\ J^{\mu}_Y &= -\left(\bar{\nu}_L\gamma^{\mu}\nu_L + \bar{\ell}_L\gamma^{\mu}\ell_L + 2\bar{\ell}_R\gamma^{\mu}\ell_R \right) \end{aligned}$$

Let us rewrite the neutral current with the new "physical" fields

$$
\begin{aligned}
W_\mu^3 &= \sin\theta_W A_\mu + \cos\theta_W Z_\mu \\
B_\mu &= \cos\theta_W A_\mu - \sin\theta_W Z_\mu
\end{aligned}
$$

where θ_W is called the Weinberg angle

Then, the neutral current will be:

$$
\begin{aligned}
\mathcal{L}_{\text{leptons}}^{(\mathsf{L+R})(0)} &= -(g\sin\theta_W J_3^\mu + \frac{1}{2}g'\cos\theta_W J_Y^\mu)A_\mu \\
&\quad +(-g\cos\theta_W J_3^\mu + \frac{1}{2}g'\sin\theta_W J_Y^\mu)Z_\mu \\
&= -g\sin\theta_W\ (\bar{\ell}\gamma^\mu\ell)\ A_\mu \\
&\quad -\frac{g}{2\cos\theta_W}\sum_{\psi_i=\nu,\ell}\bar{\psi}_i\gamma^\mu(g_V^i - g_A^i\gamma_5)\psi_i Z_\mu
\end{aligned}
$$

And $e = g\sin\theta_W = g'\cos\theta_W$

The neutral current

$$\mathcal{L}_{\rm NC}^{Z} = \frac{e}{2\sin\theta_W \cos\theta_W} Z_\mu \sum_f \bar{f}\gamma^\mu (g_V^i - g_A^i \gamma_5) f ,$$

	u	d	ν_e	e
$2\,g_V^i$	$1 - \frac{8}{3}\sin^2\theta_W$	$-1 + \frac{4}{3}\sin^2\theta_W$	1	$-1 + 4\sin^2\theta_W$
$2\,g_A^i$	1	-1	1	-1

La sección eficaz σ es el número de partículas dispersadas en un estado final específico por unidad de tiempo, dividida por el flujo inicial. La expresión para la sección eficaz diferencial es:

$$\text{Cross section} = \frac{W_{fi}}{(\text{initial flux})}(\text{number of final states})$$

This is the connection with the experiment:

A detector only sees events, thus, experimental events and theoretical events are Related by the cross section.

For a AB->CD scattering:

$$\left.\frac{d\sigma}{d\Omega}\right|_{cm} = \frac{1}{64\pi^2 s}\frac{p_f}{p_i}|\mathfrak{M}|^2$$

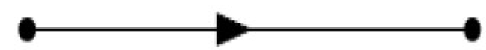
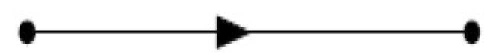
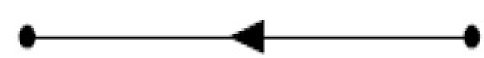
Invariant Amplitude $\mathfrak{M}$

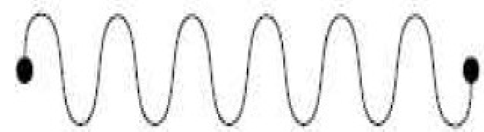
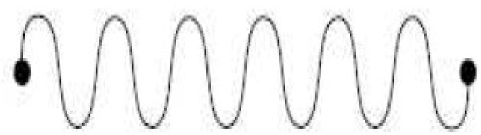
Feynman rules for computing the invariant amplitudes

A) By using the quantum number conservation, write the possible reactions

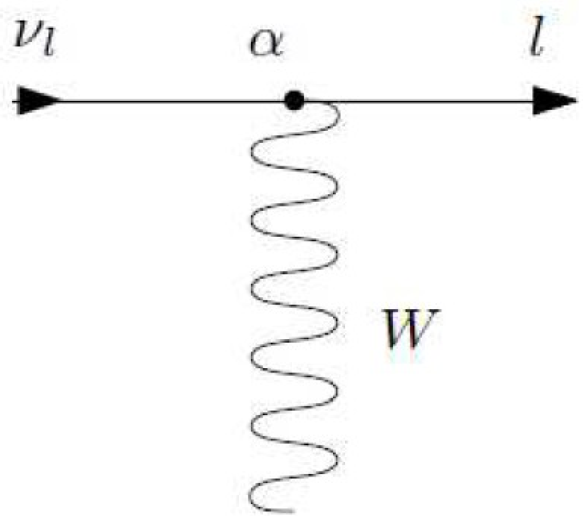
B) By looking at your Lagrangian, write the particles that mediate such reaction

C) Draw you Feynman diagram for such reaction

EXTERNAL LINES	SYMBOL	FACTOR IN .
Initial lepton line l^- or ν_l	(p, r) 	$u_r(\mathbf{p})$
Initial lepton line l^+ or $\overline{\nu}_l$	(p, r) 	$\overline{v}_r(\mathbf{p})$
Final lepton line l^- or ν_l	(p, r) 	$\overline{u}_r(\mathbf{p})$
Final lepton line l^+ or $\overline{\nu}_l$	(p, r) 	$v_r(\mathbf{p})$

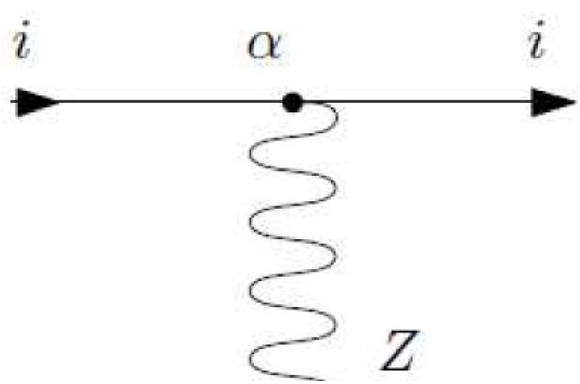
PROPAGATORS	SYMBOL	FACTOR IN $\mathcal{M}$
Internal W boson	k 	$\frac{i(-g_{\alpha\beta}+k_\alpha k_\beta/m_W^2)}{k^2-m_W^2+i\epsilon}$
Internal Z boson	k 	$\frac{i(-g_{\alpha\beta}+k_\alpha k_\beta/m_Z^2)}{k^2-m_Z^2+i\epsilon}$

SYMBOL | **FACTOR IN $\mathcal{M}$**



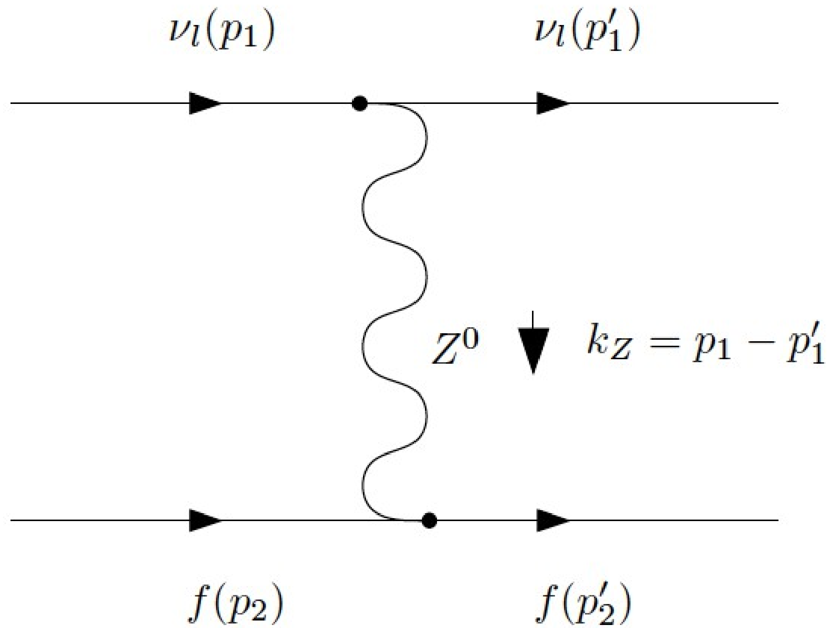
$$\frac{-\mathrm{i}g}{2\sqrt{2}}\gamma^{\alpha}(1-\gamma_5)$$

SYMBOL | **FACTOR IN $\mathcal{M}$**



$$\frac{-\mathrm{i}g}{2\cos\theta_W}\gamma^{\alpha}(g_V^i - g_A^i\gamma_5)$$

Lowest contributing order Feynman diagrams for $\nu_l + f \to \nu_l + f$.



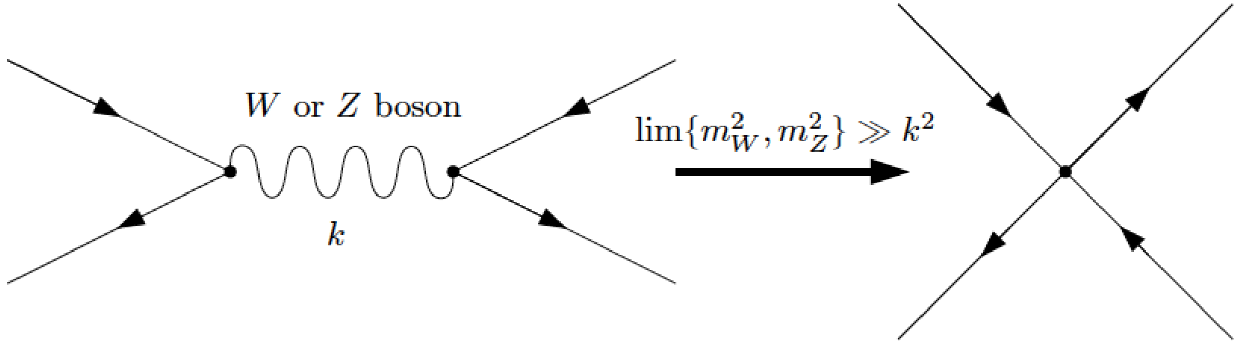
$$\mathcal{M} = \overline{u}_{\nu_l}(1')[-\frac{\mathrm{i}g}{2\cos\theta_W}\gamma^\alpha(g_V^{\nu_l} - g_A^{\nu_l}\gamma_5)]u_{\nu_f}(1)\mathrm{i}\Big(\frac{-g_{\alpha\beta} + k_{Z\alpha}k_{Z\beta}/m_Z^2}{k_Z^2 - m_Z^2 + \mathrm{i}\epsilon}\Big)$$

$$\times\ \overline{u}_f(2')[-\frac{\mathrm{i}g\gamma^\beta}{2\cos\theta_W}(g_V^f - g_A^f\gamma_5)]u_f(2,)$$

Low energy limit:

$$\lim_{m_W^2 \gg k^2} \left[\frac{-\mathrm{i}(g_{\mu\nu} - k_\mu k_\nu / m_W^2)}{k^2 - m_W^2 + \mathrm{i}\epsilon} \right] = \frac{\mathrm{i} g_{\mu\nu}}{m_W^2}$$

$$\lim_{m_Z^2 \gg k^2} \left[\frac{-\mathrm{i}(g_{\mu\nu} - k_\mu k_\nu / m_Z^2)}{k^2 - m_Z^2 + \mathrm{i}\epsilon} \right] = \frac{\mathrm{i} g_{\mu\nu}}{m_Z^2},$$



$$|\mathcal{M}|^2 \to \frac{1}{2}\sum_{s_i,s'_f}|\mathcal{M}|^2$$

$$\frac{1}{2}\sum_{s_i,s'_f}|\mathcal{M}|^2 = \frac{1}{2}\sum_{s_i,s'_f}\left[\frac{-2\mathrm{i}G_F}{\sqrt{2}}\overline{u}_{\nu_l}(1')\gamma^\alpha(g_V^{\nu_l} - g_A^{\nu_l}\gamma_5)u_{\nu_f}(1)\overline{u}_f(2')\gamma_\alpha(g_V^f - g_A^f\gamma_5)u_f(2)\right]$$
$$\times\left[\frac{2\mathrm{i}G_F}{\sqrt{2}}\overline{u}_f(2)\gamma_\beta(g_V^f - g_A^f\gamma_5)u_f(2')\overline{u}_{\nu_l}(1)\gamma^\beta(g_V^{\nu_l} - g_A^{\nu_l}\gamma_5)u_{\nu_l}(1')\right]$$

There are Dirac spinors:

$$\sum_s u_{l\alpha}(p,s)\overline{u}_{l\beta}(p,s) = (\not{p} + m_l)_{\alpha\beta}, \quad \sum_s v_{l\alpha}(p,s)\overline{v}_{l\beta}(p,s) = (\not{p} - m_l)_{\alpha\beta}.$$

$$\sum_{s_i,s'_f}|\mathcal{M}|^2 = G_F^2\sum_{s_i,s'_f}\left[\overline{u}_{\nu_l\kappa}(1')\left[\gamma^\alpha(g_V^{\nu_l} - g_A^{\nu_l}\gamma_5)\right]_{\kappa\lambda}u_{\nu_l\lambda}(1)\overline{u}_{f\gamma}(2')\left[\gamma_\alpha(g_V^f - g_A^f\gamma_5)\right]_{\gamma\delta}u_{f\delta}(2)\right]$$
$$\times\left[\overline{u}_{f\epsilon}(2)\left[\gamma_\beta(g_V^f - g_A^f\gamma_5)\right]_{\epsilon\tau}u_{f\tau}(2')\overline{u}_{\nu_l\sigma}(1)\left[\gamma^\beta(g_V^{\nu_l} - g_A^{\nu_l}\gamma_5)\right]_{\sigma\rho}u_{\nu_l\rho}(1')\right]$$

$$|\mathcal{M}|^2 \to \frac{1}{2} \sum_{s_i, s'_f} |\mathcal{M}|^2$$

$$\frac{1}{2} \sum_{s_i, s'_f} |\mathcal{M}|^2 = \frac{1}{2} \sum_{s_i, s'_f} \left[\frac{-2\mathrm{i}G_F}{\sqrt{2}} \overline{u}_{\nu_l}(1')\gamma^\alpha (g_V^{\nu_l} - g_A^{\nu_l}\gamma_5) u_{\nu_f}(1) \overline{u}_f(2')\gamma_\alpha (g_V^f - g_A^f \gamma_5) u_f(2) \right]$$

$$\times \left[\frac{2\mathrm{i}G_F}{\sqrt{2}} \overline{u}_f(2)\gamma_\beta (g_V^f - g_A^f\gamma_5) u_f(2') \overline{u}_{\nu_l}(1)\gamma^\beta (g_V^{\nu_l} - g_A^{\nu_l}\gamma_5) u_{\nu_l}(1') \right]$$

There are Dirac spinors:

$$\sum_s u_{l\alpha}(p,s)\overline{u}_{l\beta}(p,s) = (\not{p} + m_l)_{\alpha\beta}, \quad \sum_s v_{l\alpha}(p,s)\overline{v}_{l\beta}(p,s) = (\not{p} - m_l)_{\alpha\beta}.$$

$$\sum_{s_i, s'_f} |\mathcal{M}|^2 = G_F^2 \sum_{s_i, s'_f} \left[\overline{u}_{\nu_l \kappa}(1') \left[\gamma^\alpha (g_V^{\nu_l} - g_A^{\nu_l}\gamma_5) \right]_{\kappa\lambda} u_{\nu_l \lambda}(1) \overline{u}_{f\gamma}(2') \left[\gamma_\alpha (g_V^f - g_A^f\gamma_5) \right]_{\gamma\delta} u_{f\delta}(2) \right]$$

$$\times \left[\overline{u}_{f\epsilon}(2) \left[\gamma_\beta (g_V^f - g_A^f\gamma_5) \right]_{\epsilon\tau} u_{f\tau}(2') \overline{u}_{\nu_l \sigma}(1) \left[\gamma^\beta (g_V^{\nu_l} - g_A^{\nu_l}\gamma_5) \right]_{\sigma\rho} u_{\nu_l \rho}(1') \right]$$

$$= G_F^2 \mathrm{Tr} \left\{ (\not{p}'_1 + m_{\nu_l})\gamma^\alpha (g_V^{\nu_l} - g_A^{\nu_l}\gamma_5)(\not{p}_1 + m_{\nu_l})\gamma^\beta (g_V^{\nu_l} - g_A^{\nu_l}\gamma_5) \right\}$$

$$\times \mathrm{Tr} \left\{ (\not{p}'_2 + m_f)\gamma_\alpha (g_V^f - g_A^f\gamma_5)(\not{p}_2 + m_f)\gamma_\beta (g_V^f - g_A^f\gamma_5) \right\}.$$

Define:

$$
① = \text{Tr}\left\{(\not{p}'_1 + m_{\nu_l})\gamma^\alpha(g_V^{\nu_l} - g_A^{\nu_l}\gamma_5)(\not{p}_1 + m_{\nu_l})\gamma^\beta(g_V^{\nu_l} - g_A^{\nu_l}\gamma_5)\right\}
$$

$$
② = \text{Tr}\left\{(\not{p}'_2 + m_f)\gamma_\alpha(g_V^f - g_A^f\gamma_5)(\not{p}_2 + m_f)\gamma_\beta(g_V^f - g_A^f\gamma_5)\right\},
$$

and

$$
\text{Tr}\{A + B + ...\} = \text{Tr}\{A\} + \text{Tr}\{B\} + ...
$$

$$
\text{Tr}\{\gamma^5\} = \text{Tr}\{\gamma^5\gamma^\alpha\} = \text{Tr}\{\gamma^5\gamma^\alpha\gamma^\beta\} = \text{Tr}\{\gamma^5\gamma^\alpha\gamma^\beta\gamma^\gamma\} = 0
$$

$$
\text{Tr}\{\gamma^5\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\delta\} = -4\mathrm{i}\epsilon^{\alpha\beta\gamma\delta}
$$

$\text{Tr}\{\gamma^\alpha\gamma^\beta...\gamma^\rho\gamma^\sigma\} = 0$ if $\gamma^\alpha\gamma^\beta...\gamma^\rho\gamma^\sigma$ is an odd number of γ-matrices,

$$
\text{Tr}(\gamma^\alpha\gamma^\beta\gamma^\delta\gamma^\eta) = 4(\eta^{\alpha\gamma}\eta^{\delta\eta} - \eta^{\alpha\delta}\eta^{\beta\eta} + \eta^{\alpha\eta}\eta^{\beta\delta})
$$

Then:

$$① = 8p'_{1\mu}p_{1\nu}\left[\mathrm{i}g_V^{\nu_l}g_A^{\nu_l}\epsilon^{\mu\alpha\nu\beta} + \frac{1}{2}[(g_V^{\nu_l})^2 + (g_A^{\nu_l})^2](\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha} - \eta^{\mu\nu}\eta^{\alpha\beta})\right]$$

$$② = 8p_2'^{\sigma}p_2^{\rho}\left[\mathrm{i}g_V^{f}g_A^{f}\epsilon_{\sigma\alpha\rho\beta} + \frac{1}{2}[(g_V^{f})^2 + (g_A^{f})^2](\eta_{\sigma\beta}\eta_{\rho\alpha} + \eta_{\sigma\alpha}\eta_{\rho\beta} - \eta_{\sigma\rho}\eta_{\alpha\beta}) + \frac{1}{2}m_f^2(g_V^2 - g_A^2)\eta_{\alpha\beta}\right]$$

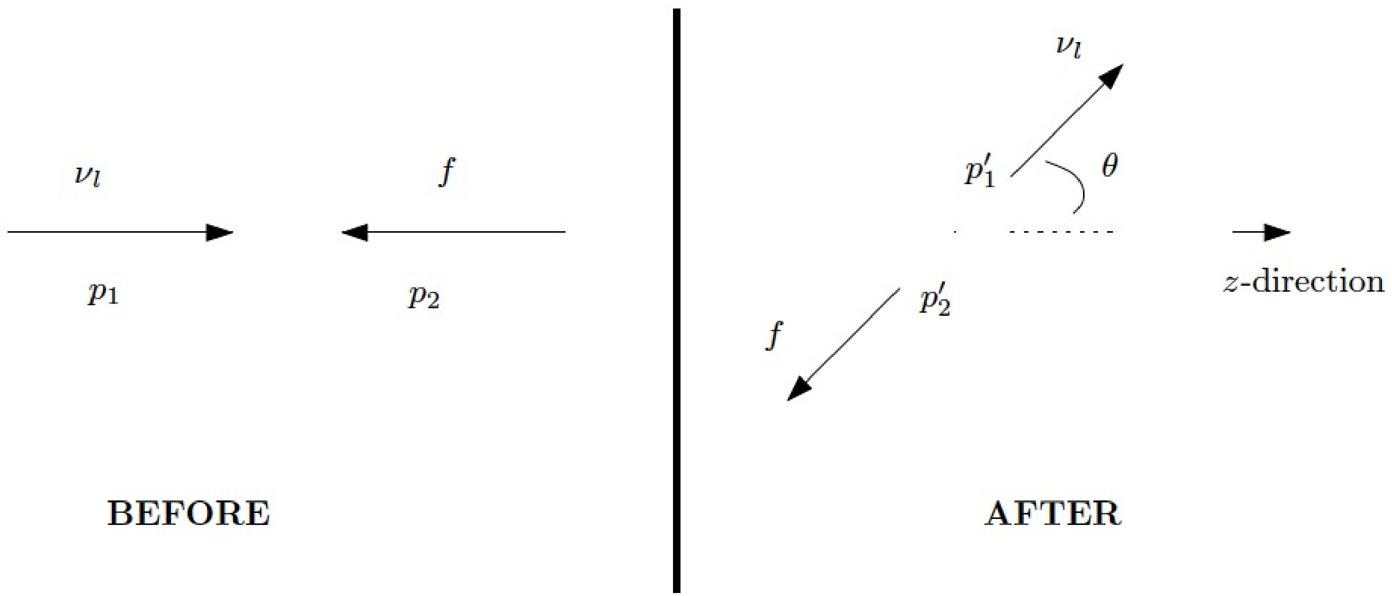
$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{CM}} = \frac{X}{64\pi^2(E_1 + E_2)^2}\frac{|\mathbf{p}_1'|}{|\mathbf{p}_1|},$$

$$X = \frac{1}{2}\sum_{s_i, s_f'}|\mathcal{M}|^2 = G_F^2[① \times ②].$$

$$\begin{aligned} ① \times ② = {} & 128 g_V^f g_A^f g_V^{\nu_l} g_A^{\nu_l}\left[(p_1'p_2')(p_1p_2) - (p_1'p_2)(p_1p_2')\right] \\ & + 32[(g_V^f)^2 + (g_A^f)^2][(g_V^{\nu_l})^2 + (g_A^{\nu_l})^2]\left[(p_1'p_2)(p_1p_2') + (p_1'p_2')(p_1p_2)\right. \\ & - 32m_f^2[(g_V^f)^2 - (g_A^f)^2][(g_V^{\nu_l})^2 + (g_A^{\nu_l})^2](p_1'p_1)\left.\right]. \end{aligned}$$

A reference system:

$$p_1 = (E, 0, 0, E),$$
$$p_2 = (\sqrt{E^2 + m_f^2}, 0, 0, -E),$$
$$p_1' = E(1, \cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta),$$
$$p_2' = (\sqrt{E^2 + m_f^2}, -E\cos\phi\sin\theta, -E\sin\phi\sin\theta, -E\cos\theta),$$



$$
\begin{aligned}
① \times ② = {} & 128 g_V^f g_A^f g_V^{\nu_l} g_A^{\nu_l} \left[(E\sqrt{E^2+m_f^2}+E^2)^2 - (E\sqrt{E^2+m_f^2}+E^2\cos\theta)^2 \right] \\
& + 32[(g_V^f)^2+(g_A^f)^2][(g_V^{\nu_l})^2+(g_A^{\nu_l})^2]\left[(E\sqrt{E^2+m_f^2}+E^2)^2+(E\sqrt{E^2+m_f^2}+E^2\cos\theta)^2\right. \\
& \left. - 32[(g_V^f)^2-(g_A^f)^2][(g_V^{\nu_l})^2+(g_A^{\nu_l})^2]E^2(1-\cos\theta)\right].
\end{aligned}
$$

Cross section is then:

$$
\sigma[\nu_l f\,] = \frac{G_F^2}{32\pi(E+\sqrt{E^2+m_f^2})^2}\int_0^\pi ① \times ② \sin\theta \mathrm{d}\theta .
$$

And finally:

$$
\sigma[\nu_l f\,] = \frac{G_F^2(s-m_f^2)^2}{4\pi s}\left[(g_V^f+g_A^f)^2+(g_V^f-g_A^f)^2\left[\frac{m_f^2}{s}+\frac{(s-m_f^2)^2}{3s^2}\right]-[(g_V^f)^2-(g_A^f)^2]\frac{m_f^2}{s}\right]
$$

$$
s = (p_1+p_2)^2 = (E+\sqrt{E^2+m_f^2})^2
$$

Ready for comparison with experiments:

Experiment	Energy	Events	Measurement, σ	$\sin^2\theta_W$
LAMPF	7-60	236	$[10{,}0 \pm 1{,}5 \pm 0{,}9] E_{\nu_e} \cdot 10^{-45}\mathrm{cm}^2$	$0{,}249 \pm 0{,}063$
LSND	10-50	191	$[10{,}1 \pm 1{,}5] \cdot E_{\nu_e} \cdot 10^{-45}\mathrm{cm}^2$	$0{,}248 \pm 0{,}051$
Irvine	1.5-3.0	381	$[0{,}86 \pm 0{,}25] \cdot \sigma_{V-A}$	$0{,}29 \pm 0{,}05$
	3.0-4.5	77	$[1{,}7 \pm 0{,}44] \cdot \sigma_{V-A}$	
Krasnoyarsk	3.15-5.175	N/A	$[4{,}5 \pm 2{,}4] \cdot 10^{-46}\mathrm{cm}^2/\mathrm{fission}$	$0{,}22^{+0{,}7}_{-0{,}8}$
Rovno	0.6-2.0	41	$[1{,}26 \pm 0{,}62] \cdot 10^{-44}\mathrm{cm}^2/\mathrm{fission}$	N/A
MUNU	0.7-2.0	68	$1{,}07 \pm 0{,}34$ events day^{-1}	N/A
Global				$0{,}259 \pm 0{,}025$

Un ejemplo arquetípico de astropartícula: Neutrinos solares

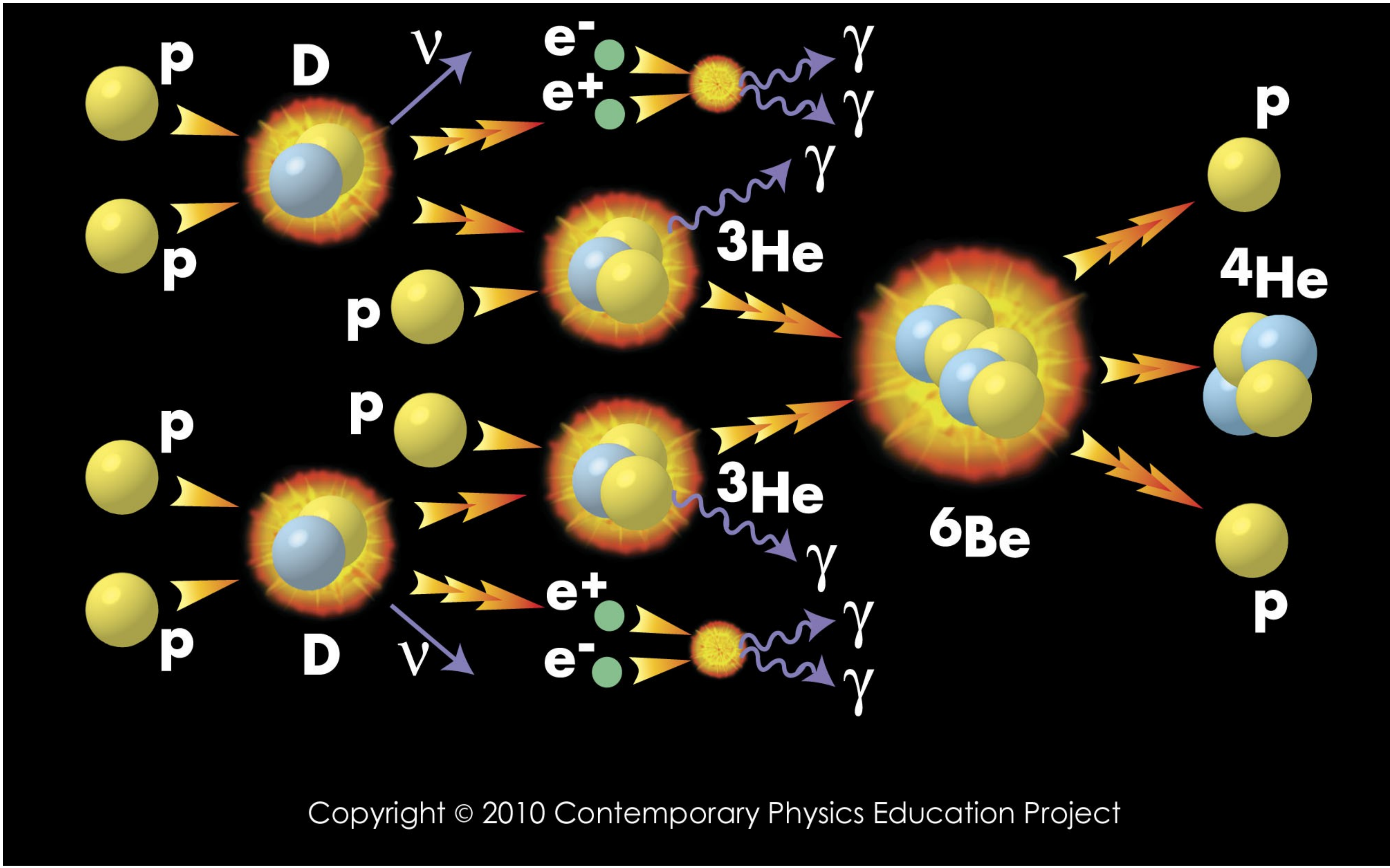
$$N_{Obs}^{th} = \sum_i \int dE_\nu \int dT \phi_i \lambda_i(E_\nu) \times \frac{d\sigma(E_\nu, T)}{dT} \times t \times N_e \times R(T, T')$$

Producción: Flujo de Neutrinos solares

Detección

Propagación: Prácticamente sin cambio (hipótesis)

El problema de los neutrinos solares



El problema de los neutrinos solares

Ray Davies

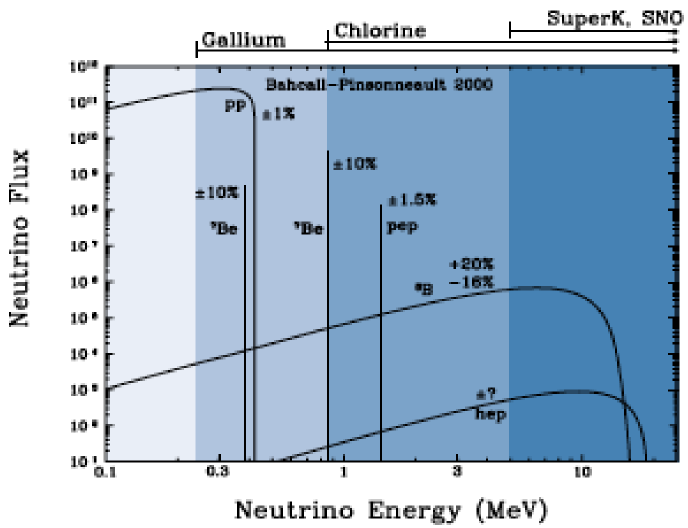
John Bacall

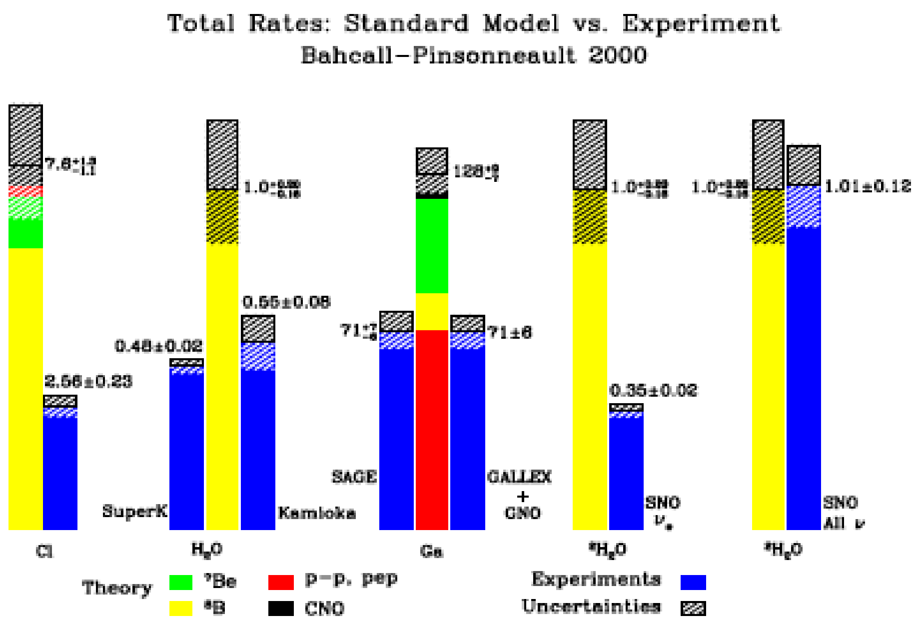
Experimento en Homestake

1970-1994

$$N^{th}_{Obs} = \sum_i \int dE_\nu \int dT \phi_i \lambda_i(E_\nu) \times \frac{d\sigma(E_\nu, T)}{dT} \times t \times N_e \times R(T, T')$$

1. Modelo estándar solar (MES): $\phi_i, \lambda_i(E_\nu)$
2. Experimento: $R(T, T'), N_e$
3. Modelo estándar electro-débil: $\frac{d\sigma(E_\nu, T)}{dT}$





¿Cómo resolver este problema?

- Incluir masa a los neutrinos y mezcla entre los auto-estados de masa
- Momento magnético grande
- Interacciones no-estándares en corrientes neutras que cambian sabor

$$N_{Obs}^{th} = \sum_i \phi_i \times t \times N_e \times \int dE_\nu \int dT \lambda_i(E_\nu) \times \frac{d\sigma(E_\nu, T)}{dT} \times P(\Delta m^2, \theta, \mu_\nu, \epsilon, \varepsilon' ...)$$

1. Oscilación de neutrinos Hipótesis: Los neutrinos tienen masa
2. Precesión espín-sabor Hipótesis: Los neutrinos tienen masa y un momento magnético μ_ν

Un fenómeno puramente cuántico

1.Oscilación de neutrinos

Hipótesis: Los neutrinos tienen masa

$$|\nu_\alpha\rangle = \sum_a U^*_{\alpha a}|\nu_a\rangle$$

$$H = \begin{pmatrix} p + \frac{m_1^2}{2p} & 0 & 0 \\ 0 & p + \frac{m_2^2}{2p} & 0 \\ 0 & 0 & p + \frac{m_3^2}{2p} \end{pmatrix}, \qquad |\nu_\alpha, t\rangle = e^{-iHt}|\nu_\alpha, t=0\rangle$$

En el vacío por ejemplo:

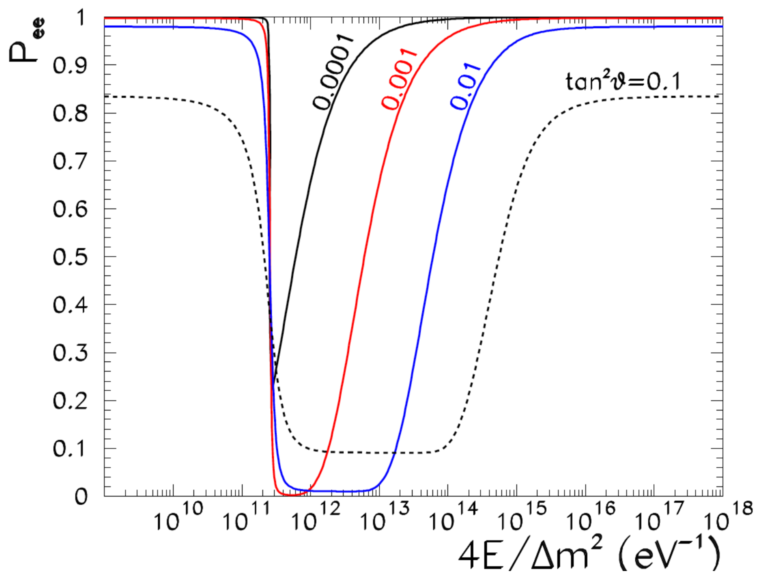
$$P(\bar{\nu}_e \to \bar{\nu}_e) \approx 1 - \text{sen}^2(2\theta_{13})\text{sen}^2\left(\frac{\Delta m^2_{23}}{4E}L\right) - \cos^4\theta_{13}\text{sen}^2(2\theta_{12})\text{sen}^2\left(\frac{\Delta m^2_{12}}{4E}L\right)$$

Interacción con la materia

- Corrientes neutras (NC): intercambio de Z_0
- Corrientes cargadas (CC): intercambio de $W_\pm$

Ecuación de evolución

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \sqrt{2}\,G_F N_e & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$



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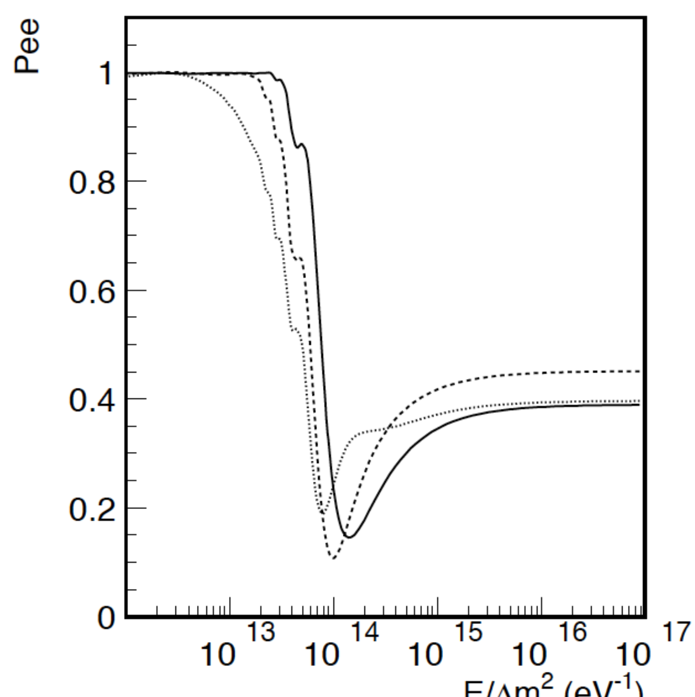
Masa vs. Momento magnético

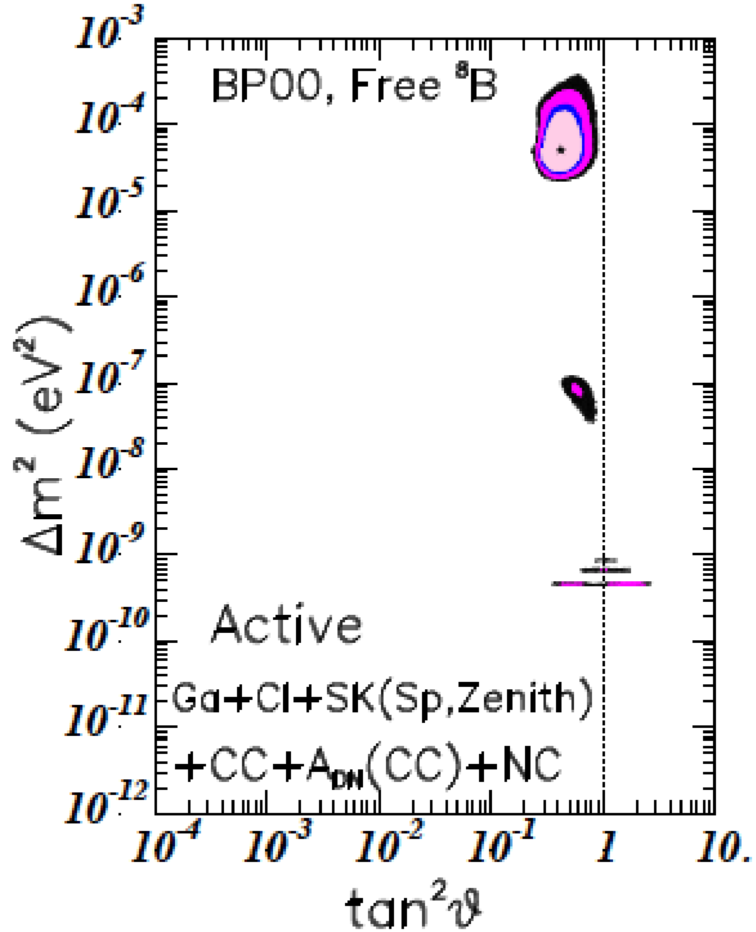
2. Precesión espín-sabor

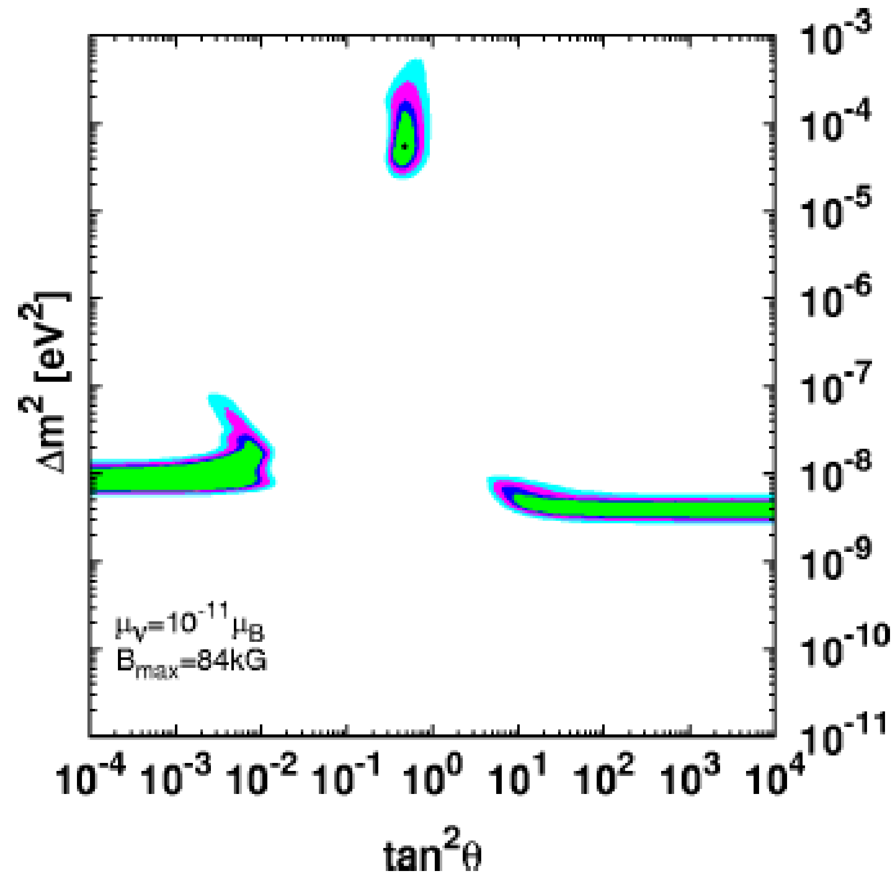
Hipótesis: Los neutrinos tienen masa y un momento magnético μ_ν

$$i \begin{pmatrix} \dot{\nu}_{eL} \\ \dot{\bar{\nu}}_{eR} \\ \dot{\nu}_{\mu L} \\ \dot{\bar{\nu}}_{\mu R} \end{pmatrix} = \begin{pmatrix} V_e - c_2\delta & 0 & s_2\delta & \mu B_+(t) \\ 0 & -V_e - c_2\delta & -\mu B_-(t) & s_2\delta \\ s_2\delta & -\mu B_+(t) & V_\mu + c_2\delta & 0 \\ \mu B_-(t) & s_2\delta & 0 & -V_\mu + c_2\delta \end{pmatrix} \begin{pmatrix} \nu_{eL} \\ \bar{\nu}_{eR} \\ \nu_{\mu L} \\ \bar{\nu}_{\mu R} \end{pmatrix},$$

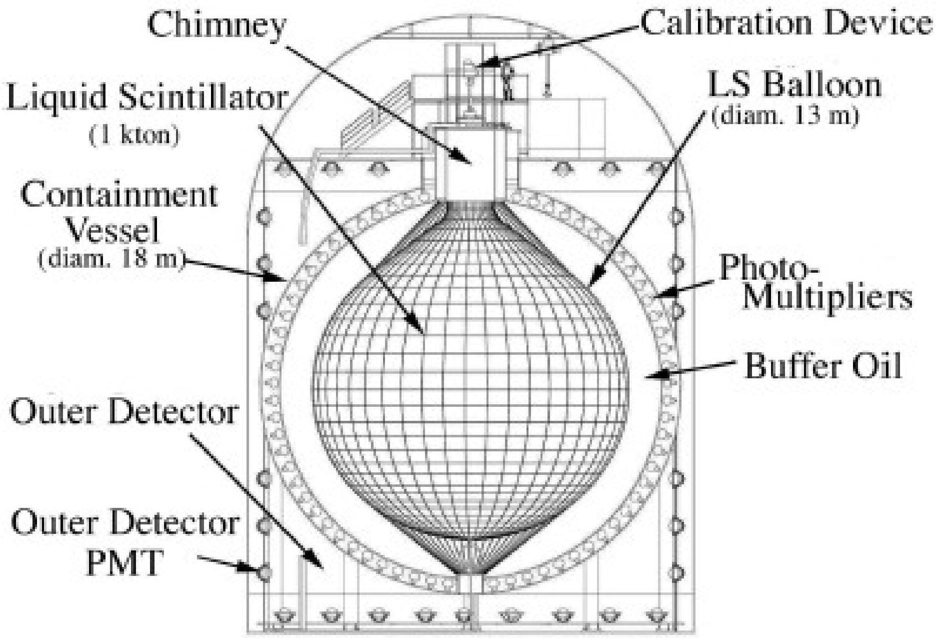


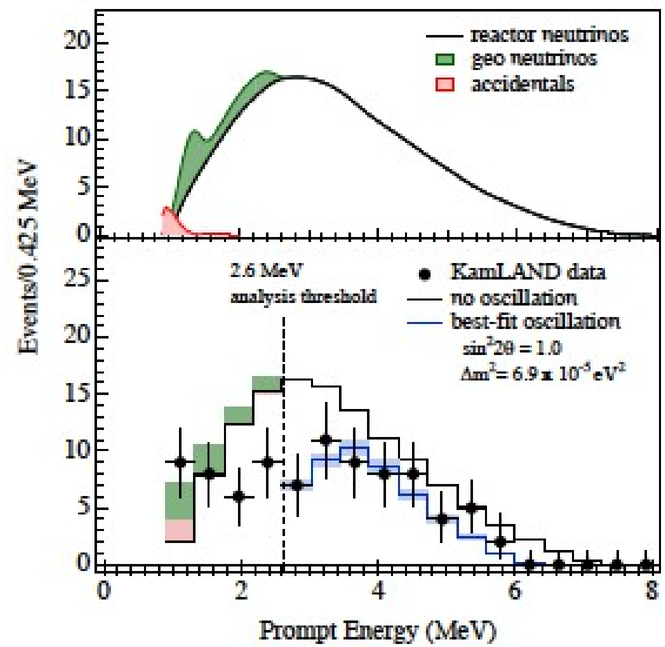
Masa vs. Momento magnético

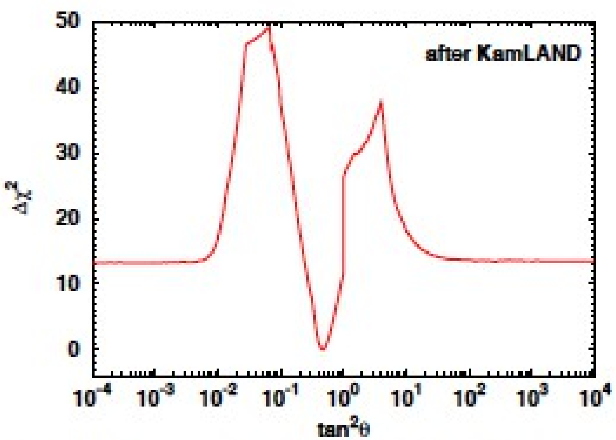




Kamland

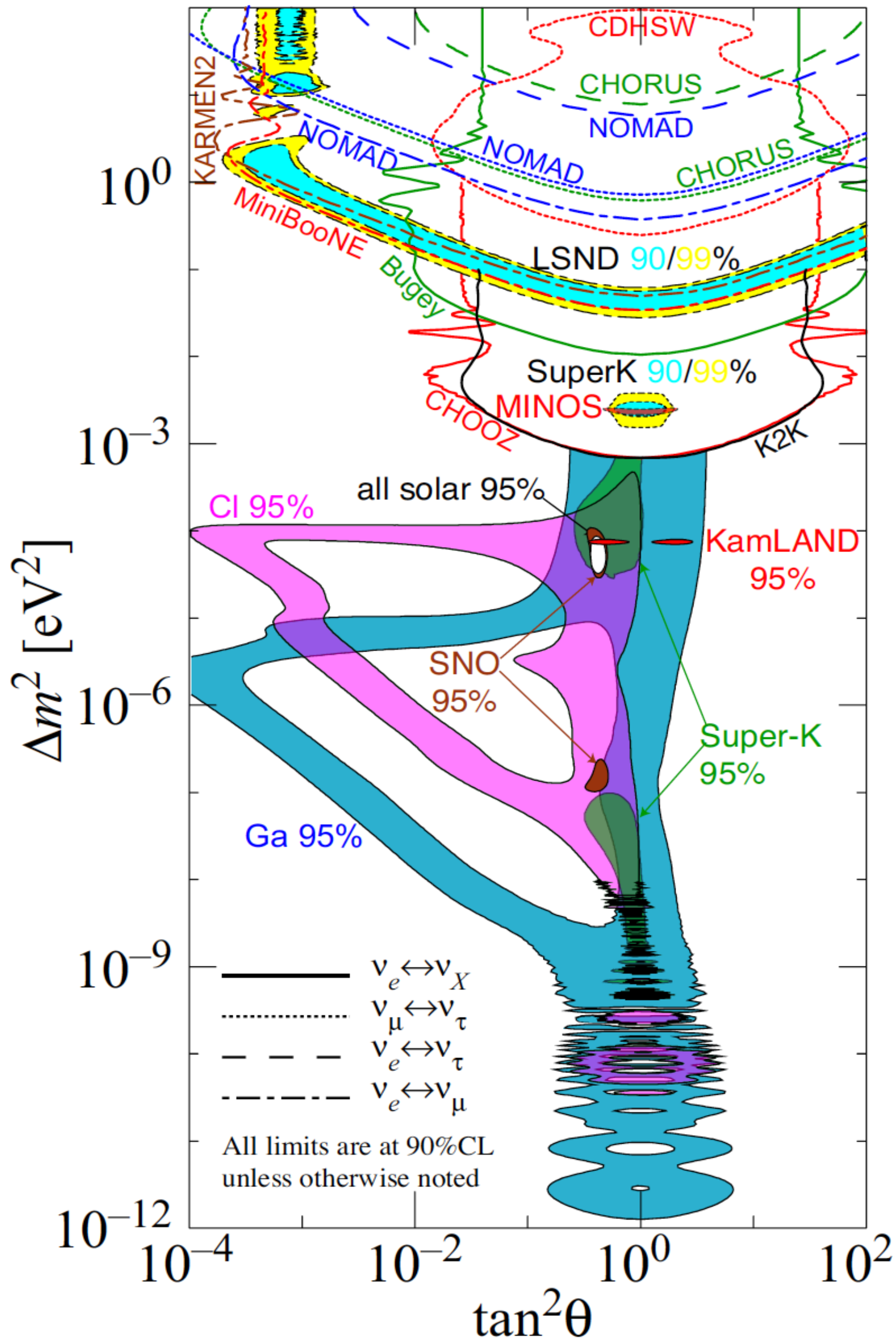






[O.G. Miranda *et al.* 2002]

Un modelo consistente



$$N_{Obs}^{th} = \sum_i \phi_i \times t \times N_e \times \int dE_\nu \int dT \lambda_i(E_\nu) \times \frac{d\sigma(E_\nu, T)}{dT} \times P(\Delta m^2, \theta, \mu_\nu, \epsilon, \varepsilon' \ldots)$$

Producción

Detección

Propagación

Continuará...