Quasinormal modes 2: Spinning black holes

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Last time

- Scalar wave equation: $\Box_g \Phi = -4\pi T$
- Separation of variables:

$$\Phi_{\omega lm} \sim e^{-i\omega t} \frac{u_{\omega lm}(r)}{r} Y_{lm}(\theta, \phi)$$

- Radial equation:

$$\frac{d^2 u_{\omega lm}}{dr_*^2} + \left(\omega^2 - V\right) u_{\omega lm} = 0$$

$$V = \left(1 - \frac{2M}{r}\right) \left(\frac{l(l+1)}{r^2} + \frac{2M}{r^3}\right)$$

Last time

- For orbiting source with a few freq

$$\frac{d^2 u_{\omega l m}}{dr_*^2} + (\omega^2 - V) u_{\omega l m} = S_{\omega l m}(r)$$

  - Decompose source
  - GF via series sln or direct integration
  - Integrate GF over source moments
  - Assemble
- Or, solve 1+1 D wave eqn with source

Last time

- Outgoing BCs only for special freq
- Leaver's method gives

$$\omega_{lmn} \qquad u_{lmn}(r) \qquad A_{\rm out}$$

- Similar series expansions get $\partial_\omega A_{\rm in}$

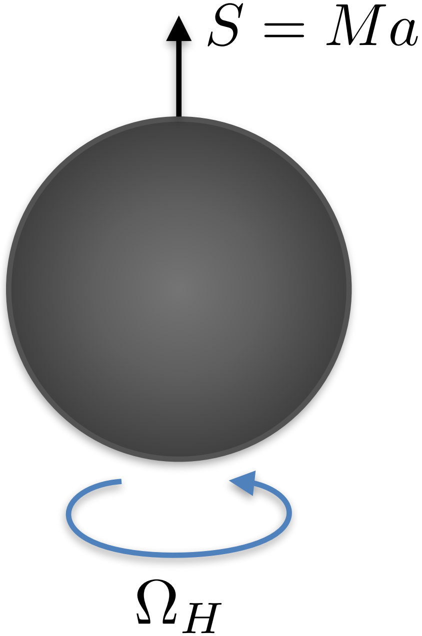
- This gives GF, use source evaluated at QNMs to get amplitudes and phases modes

- Grav waves treated the same: ${}_s\psi_{\omega lm} \to h_{\mu\nu}$

Part 1

ROTATING BLACK HOLES

Rotating black holes: Kerr

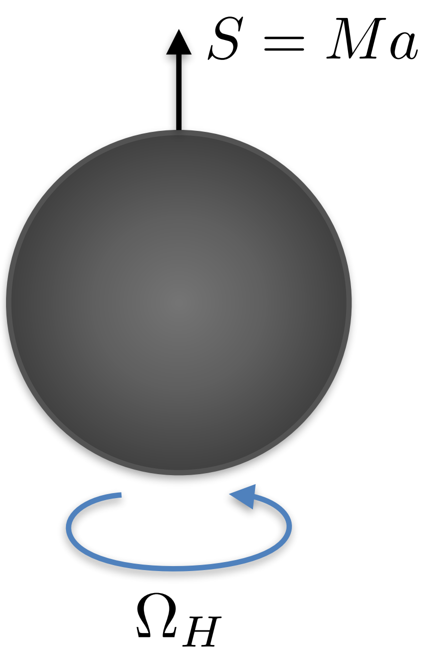


- Parametrized by mass and spin parameter $M,\ a$
- Horizon at $r_+ = M(1 + \sqrt{1 - a^2/M^2})$
- Frame dragging: $g_{t\phi} \neq 0$
- Ergoregion

$$r^2 - 2Mr + a^2 \cos^2\theta < 0$$

- Horizon rotates at $\Omega_H \propto a$

Rotating black holes: Kerr



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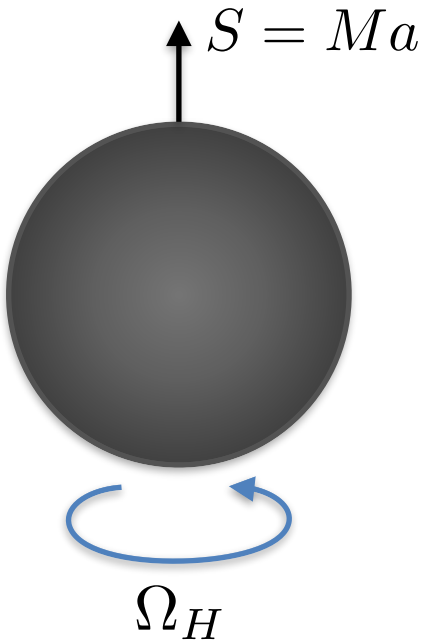
$$r^2 - 2Mr + a^2\cos^2\theta < 0$$

- Horizon rotates at $\Omega_H \propto a$

$$g_{\mu\nu}dx^\mu dx^\nu = -\left(1 - \frac{2Mr}{r^2 + a^2\cos^2\theta}\right)dt^2 + \frac{r^2 + a^2\cos^2\theta}{\Delta}dr^2 + (r^2 + a^2\cos^2\theta)d\theta^2$$

$$-\frac{4Mar\sin^2\theta}{r^2 + a^2\cos^2\theta}dt d\phi + \frac{\sin^2\theta((r^2 + a^2)^2 - a^2\Delta\sin^2\theta}{r^2 + a^2\cos^2\theta}d\phi^2$$

Rotating black holes: Kerr



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- Ergoregion

$$r^2 - 2Mr + a^2\cos^2\theta < 0$$

- Horizon rotates at $\Omega_H \propto a$

$$\Delta = r^2 - 2Mr + a^2 = (r-r_+)(r-r_-)$$

Weyl curvature scalar

- For Kerr spacetime, curvature quantities most natural

$$\Psi_4 = C_{\mu\nu\rho\sigma} n^\mu m^{\nu *} n^\rho m^{\sigma *}$$

- Example: flat space perts $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$l^\mu = \frac{(\partial_t)^\mu + (\partial_r)^\mu}{\sqrt{2}} \qquad n^\mu = \frac{(\partial_t)^\mu - (\partial_r)^\mu}{\sqrt{2}}$$

$$m^\mu = \frac{(\partial_\theta)^\mu + i \csc\theta (\partial_\phi)^\mu}{\sqrt{2} r}$$

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$$\Psi_4 = -\ddot{h}_+ + i\ddot{h}_\times$$

Perturbations of Kerr

$$\begin{array}{ccccc} \Phi & & \Phi & & \\ A_\mu & \longrightarrow & \phi_0, \phi_2 & \longrightarrow & L_T[{}_s\psi] = 0 \\ h_{\mu\nu} & & \Psi_0, \Psi_4 & & \end{array}$$

Project onto basis

$l^\mu, n^\mu, m^\mu, m^{*\mu}$

Master equation

$${}_s\psi_{lm\omega} = e^{-i\omega t + im\phi}\,{}_sR_{lm\omega}(r)\,{}_sS_{lm\omega}(\theta)$$

$$s = -2:\ {}_s\psi = \Psi_4 \rightarrow h_{\mu\nu}$$

Black hole perturbation theory

- Angular equation

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\,S_{lm\omega}}{d\theta}\right)+V_\theta(\omega,A_{lm})S_{lm\omega}=0$$

- Potential is more complicated than Schw

$$V_\theta = {}_sE_{lm\omega}-\frac{m^2}{\sin^2\theta}-s^2\cot^2\theta-s^2+a^2\omega^2\cos^2\theta-2a\omega s\cos\theta$$

- Solutions are deformations of (spin-weighted) spherical harmonics

Black hole perturbation theory

- Radial equation for $u_{lm\omega} = \Delta^{s/2}\sqrt{r^2 + a^2}R_{lm\omega}$

$$\frac{d^2 u_{lm\omega}}{dr_*^2} + V_r\, u_{lm\omega} = S_{lm\omega}(r)$$

$$V_r = \left(\omega - \frac{am}{r^2 + a^2}\right)^2 - 2is\frac{r - M}{r^2 + a^2}\left(\omega - \frac{am}{r^2 + a^2}\right) + \frac{\Delta}{r^2 + a^2)^2}(4ir\omega s - \lambda) + F(r, s)$$

$${}_s\lambda_{lm\omega} = {}_sE_{lm\omega} - s(s+1) + a^2\omega^2 - 2am\omega$$

Black hole perturbation theory

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$$r_* = r + \frac{r_+^2+a^2}{r_+-r_-}\ln\left(\frac{r-r_+}{r_+}\right) - \frac{r_-^2+a^2}{r_+-r_-}\ln\left(\frac{r-r_-}{r_+}\right)$$

Scattering and superradiance

- Scattering of scalar waves as before

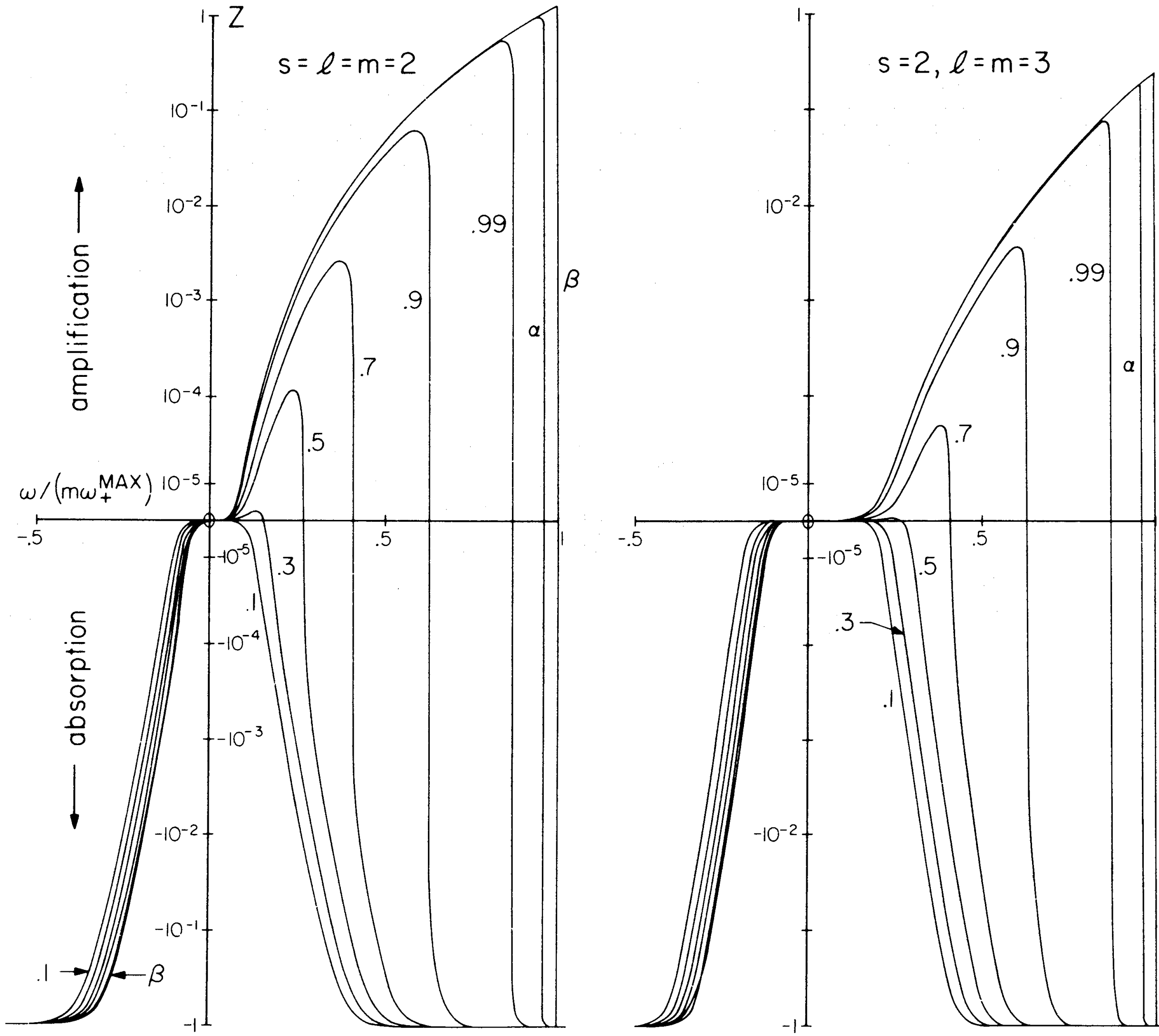
$$u_{\rm in} \sim \begin{cases} A_{\rm in} e^{-i\omega r_*} + A_{\rm out} e^{i\omega r_*} & r_* \to \infty \\ e^{-i(\omega - m\Omega_H) r_*} & r_* \to -\infty \end{cases}$$

- Try to derive flux conservation

$$W[u_{\rm in}, u_{\rm in}^*] = u_{\rm in} \frac{du_{\rm in}^*}{dr_*} - u_{\rm in}^* \frac{du_{\rm in}}{dr_*} \Rightarrow \left(1 - \frac{m\Omega_H}{\omega}\right) |\mathcal{T}|^2 = 1 - |\mathcal{R}|^2$$

- More flux out than went in if $\omega < m\Omega_H$
- Energy taken from BH: Penrose process for waves

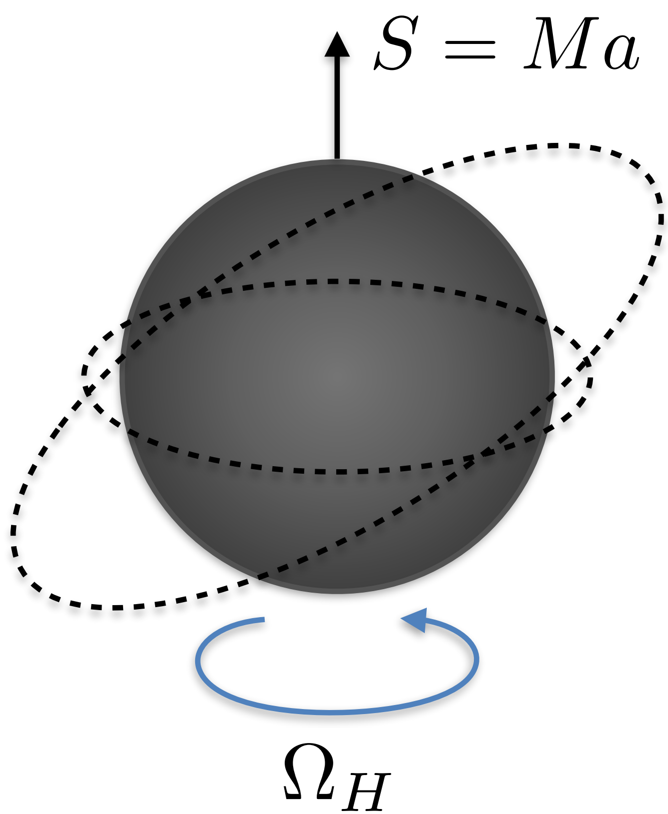
Superradiance (Grav scattering)



$$Z = \frac{dE_{\rm out}/dt}{dE_{\rm in}/dt}$$

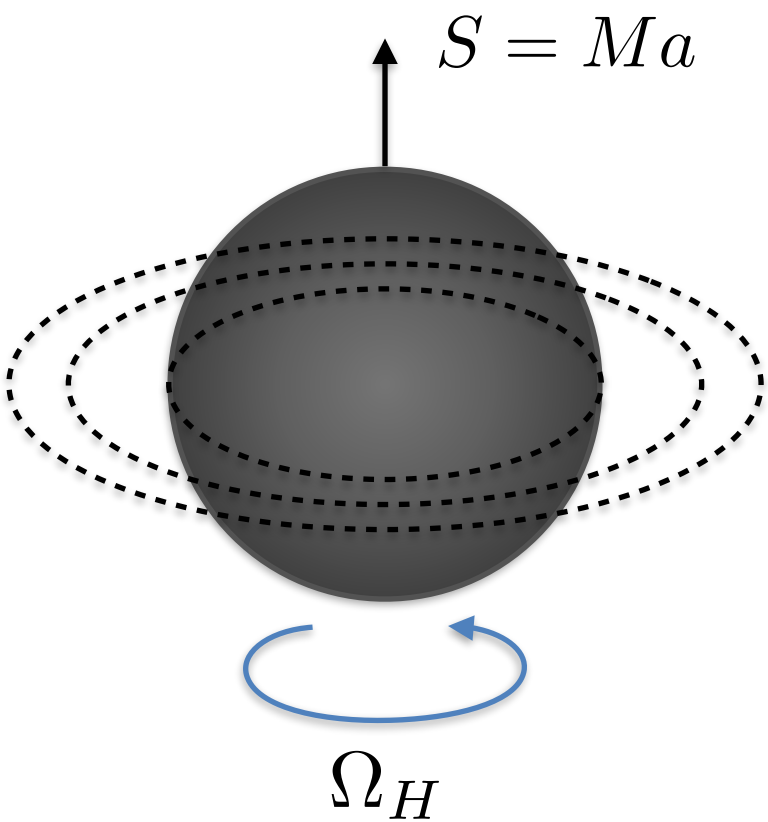
Teukolsky & Press (1974)

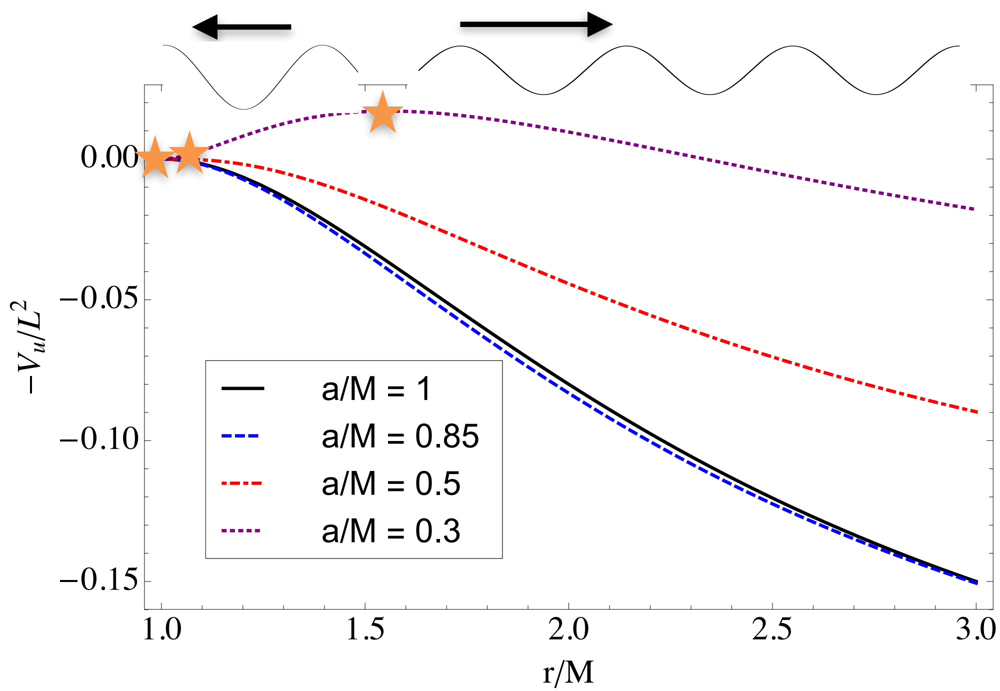
QNMs of Kerr



- Photon orbits vary with inclination
- Wave picture: freq and decay split with m
- As spin increases, freq increases, decay rate decreases
- Modes determined by mass and spin

Geometric optics (WKB)





Yang et al (2012)

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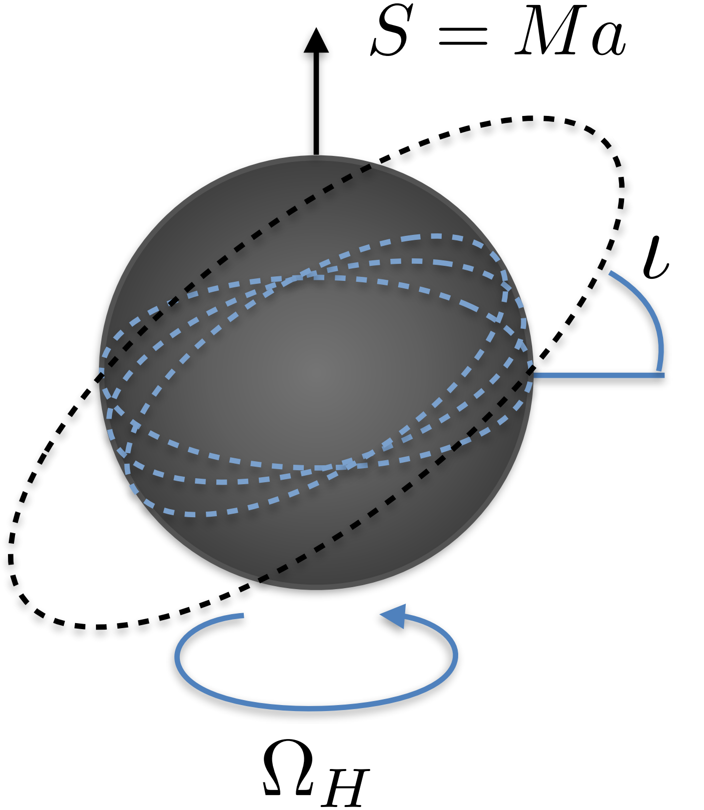
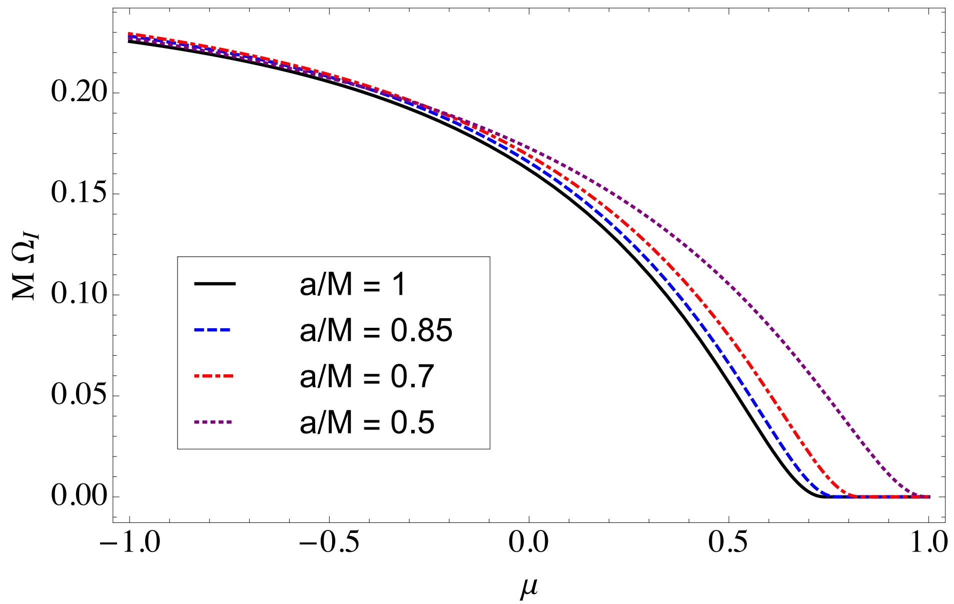


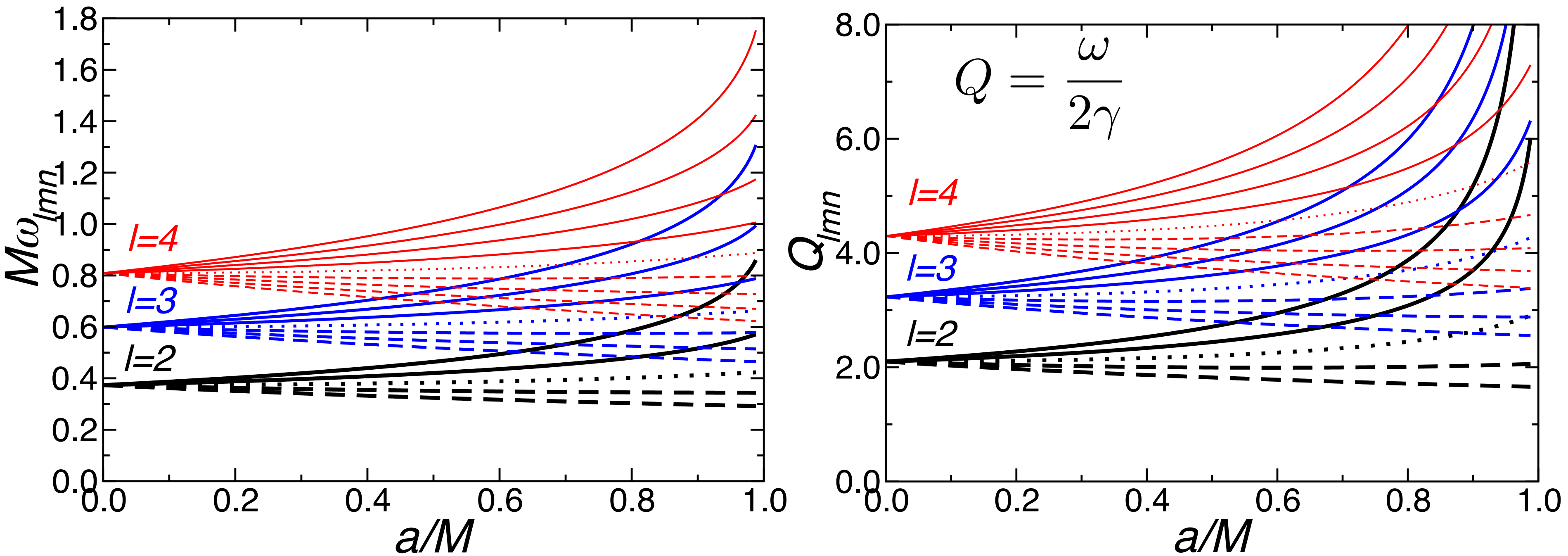


$$\mu \sim \cos \iota$$

$$\iota \sim 42^\circ$$

Yang et al (2012)

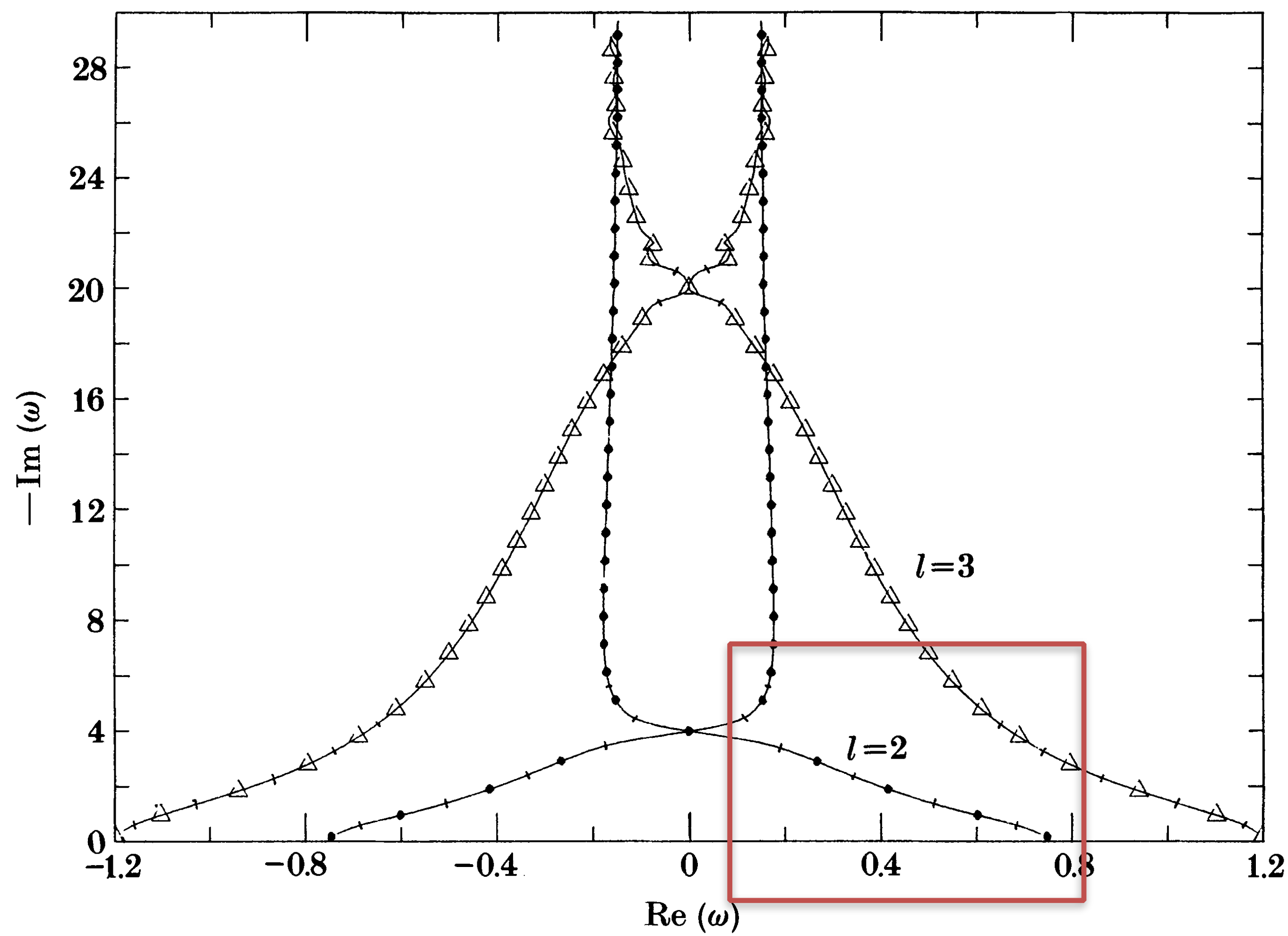
Kerr QNMs



$$(\omega_R, \gamma) \rightarrow (M, a/M)$$

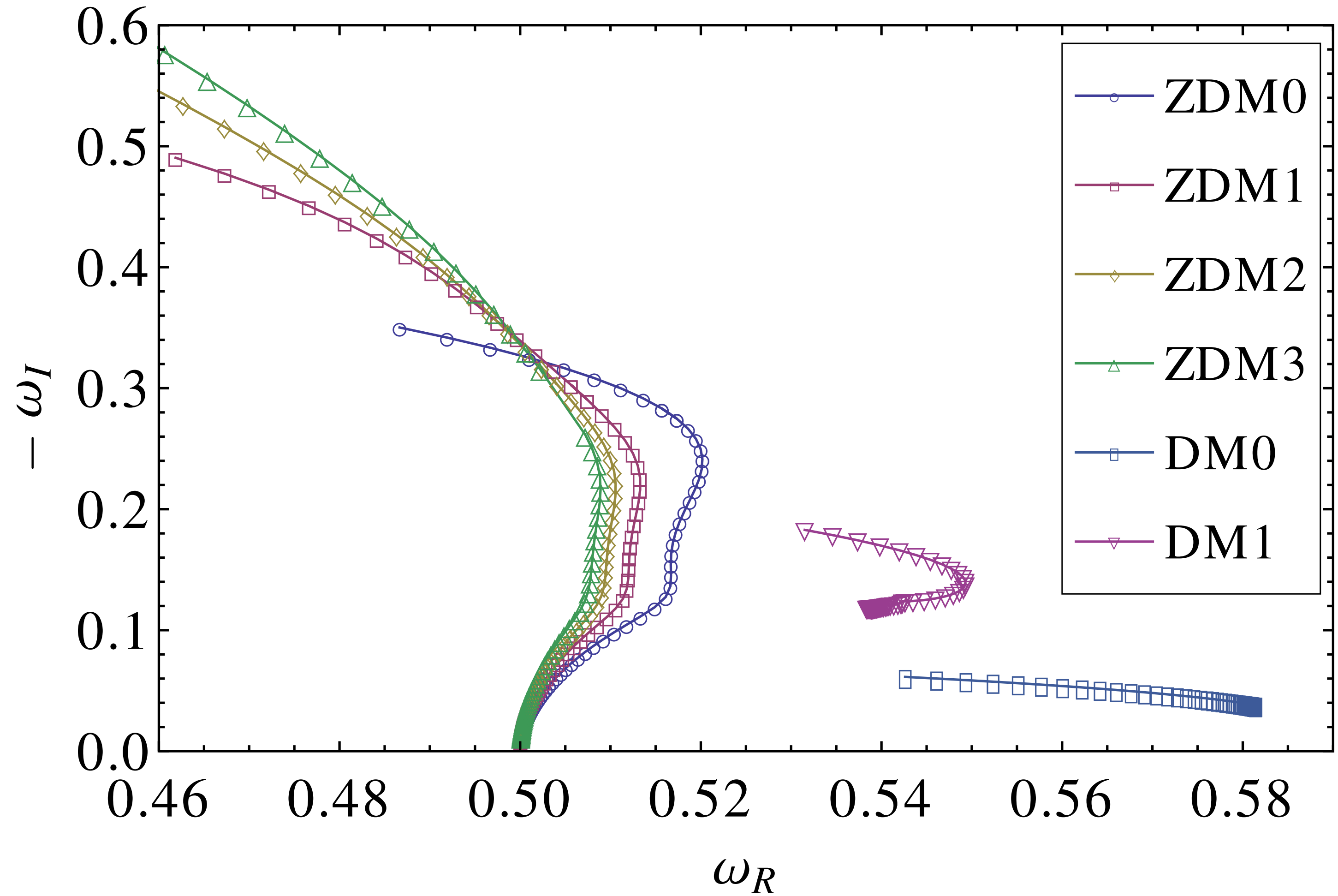
Berti,Cardoso, Starinets (2009)

Schwarzschild QNMs



Leaver (1985)

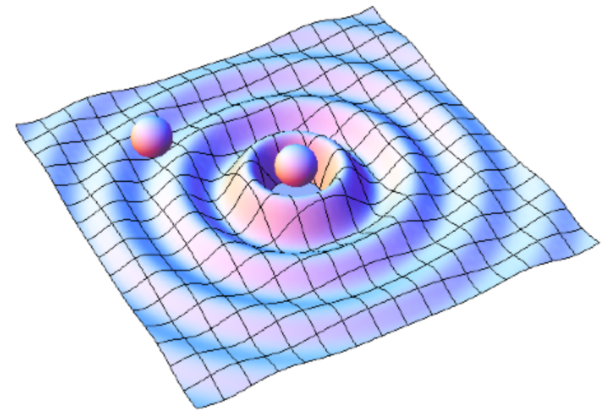
Splitting of spectrum



Yang, AZ et al. (2013)

Quasinormal mode response

- Have source-free solutions



$$\psi_{lm\omega} \sim e^{-i\omega t + im\phi} R_{lm\omega}(r) S_{lm\omega}(\theta)$$

$$R_{lm\omega} = \Delta^{-s/2}(r^2 + a^2)^{-1/2} u_{lm\omega}$$

- Build response func in time domain

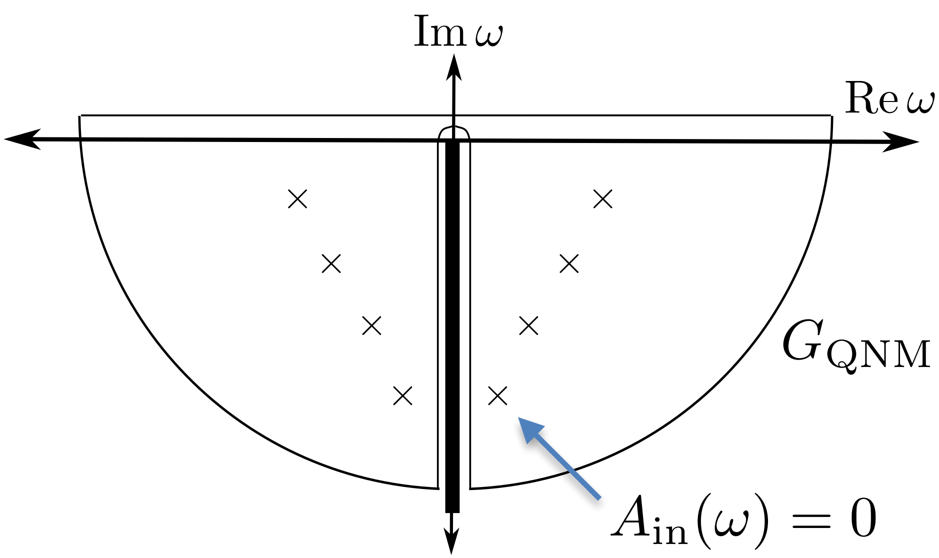
$$G(x^\mu, x^{\mu\prime}) = \frac{1}{2\pi} \sum_{l,m} \int d\omega \, e^{-i\omega(t-t')} g_{lm\omega}(r, r') \Omega(\theta, \theta', \phi, \phi')$$

$$g(r, r') = \Delta'^{s} \frac{R^{\rm in}_{lm\omega}(r') R^{\rm up}_{lm\omega}(r)}{2i\omega A_{\rm in}}$$

Quasinormal mode response

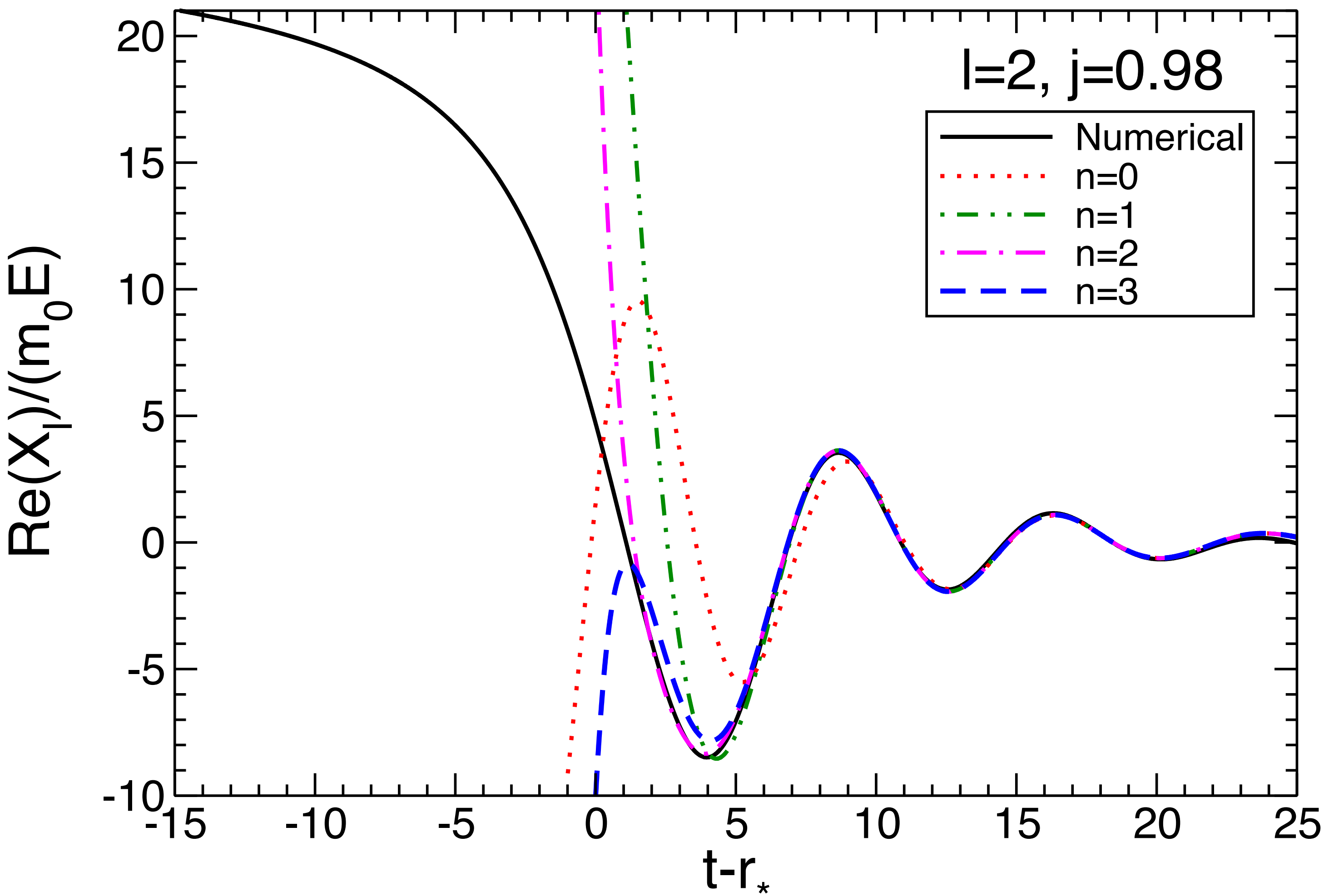
- Time domain response from inverse Laplace transform

$$G(x^\mu, x^{\mu\prime}) \sim \sum_{l,m} \int d\omega \, e^{-i\omega(t-t')} \Delta'^s \frac{R^{\text{in}}_{lm\omega}(r') R^{\text{up}}_{lm\omega}(r)}{2i\omega A_{\text{in}}}$$



$$G_{\text{QNM}} \sim \sum_{lmn} e^{-i\omega_n(t-t')} \frac{\Delta'^s R^{\text{in}}_{lmn}(r') R^{\text{up}}_{lmn}(r)}{\omega_n \partial_\omega A_{\text{in}}|_{\omega_n}}$$

Ringdown and QNMs

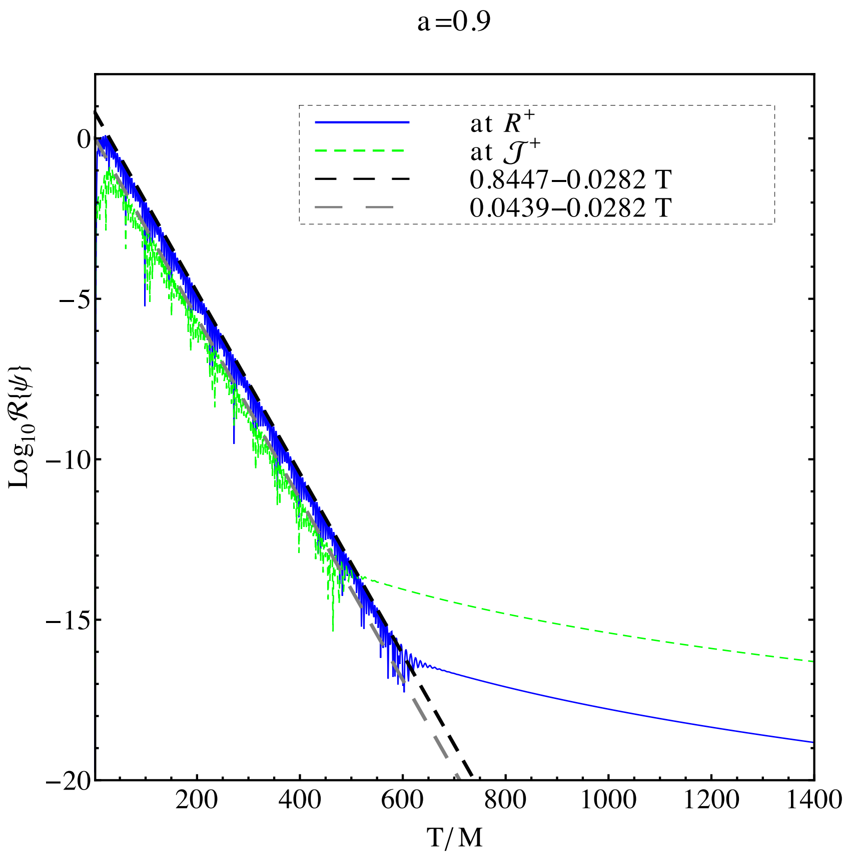


Zhang, Berti, Cardoso (2013)

Late-time tail radiation

- Branch cut associated with power-law decay
- Seen at very late times
- Physically, due to backscatter off long-range potential

$$G_{\mathrm{BC}} \propto \frac{(r_* r'_*)^{l+1}}{(t-t')^{2l+3}}$$

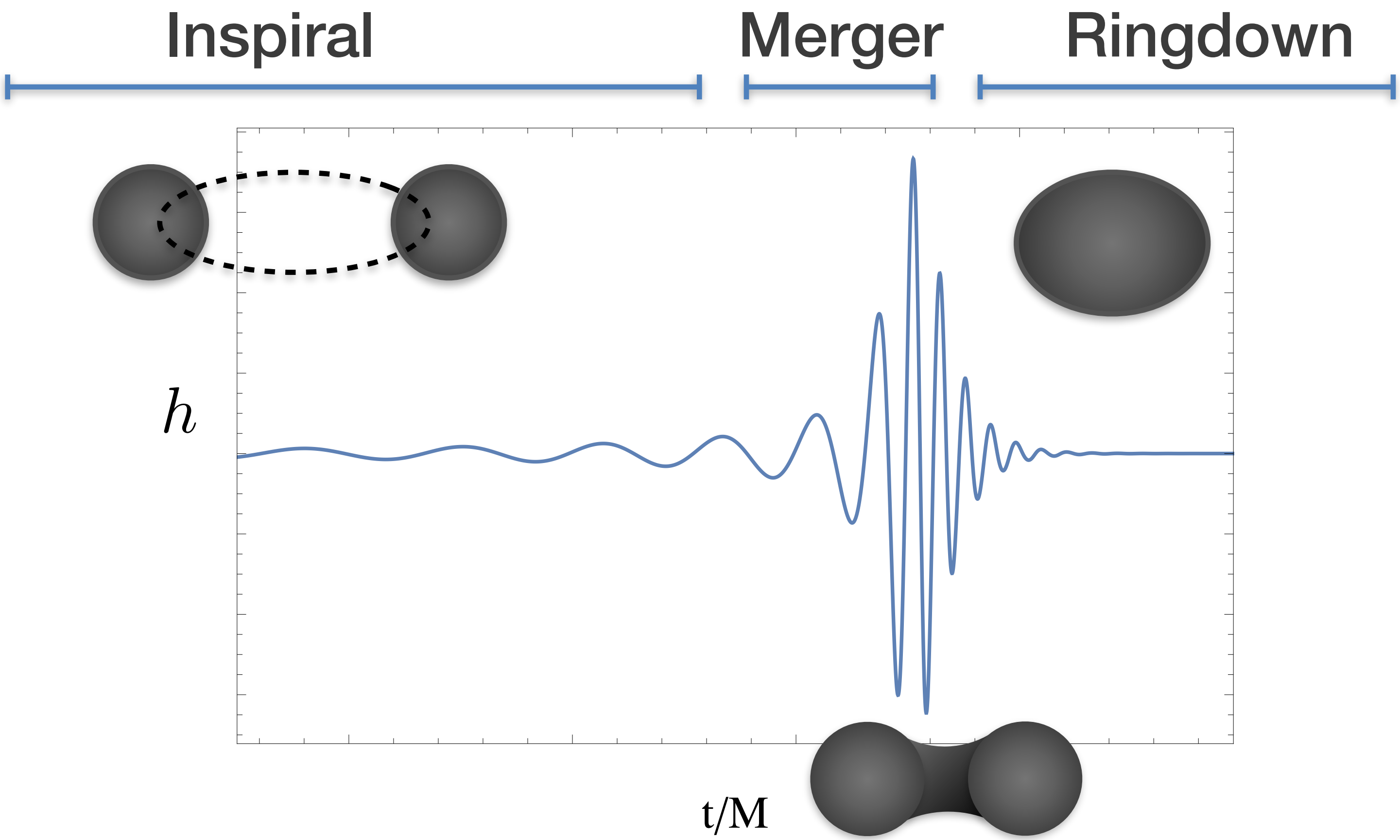


Harms, Bernuzzi, Brugmann (2012)

Part 2

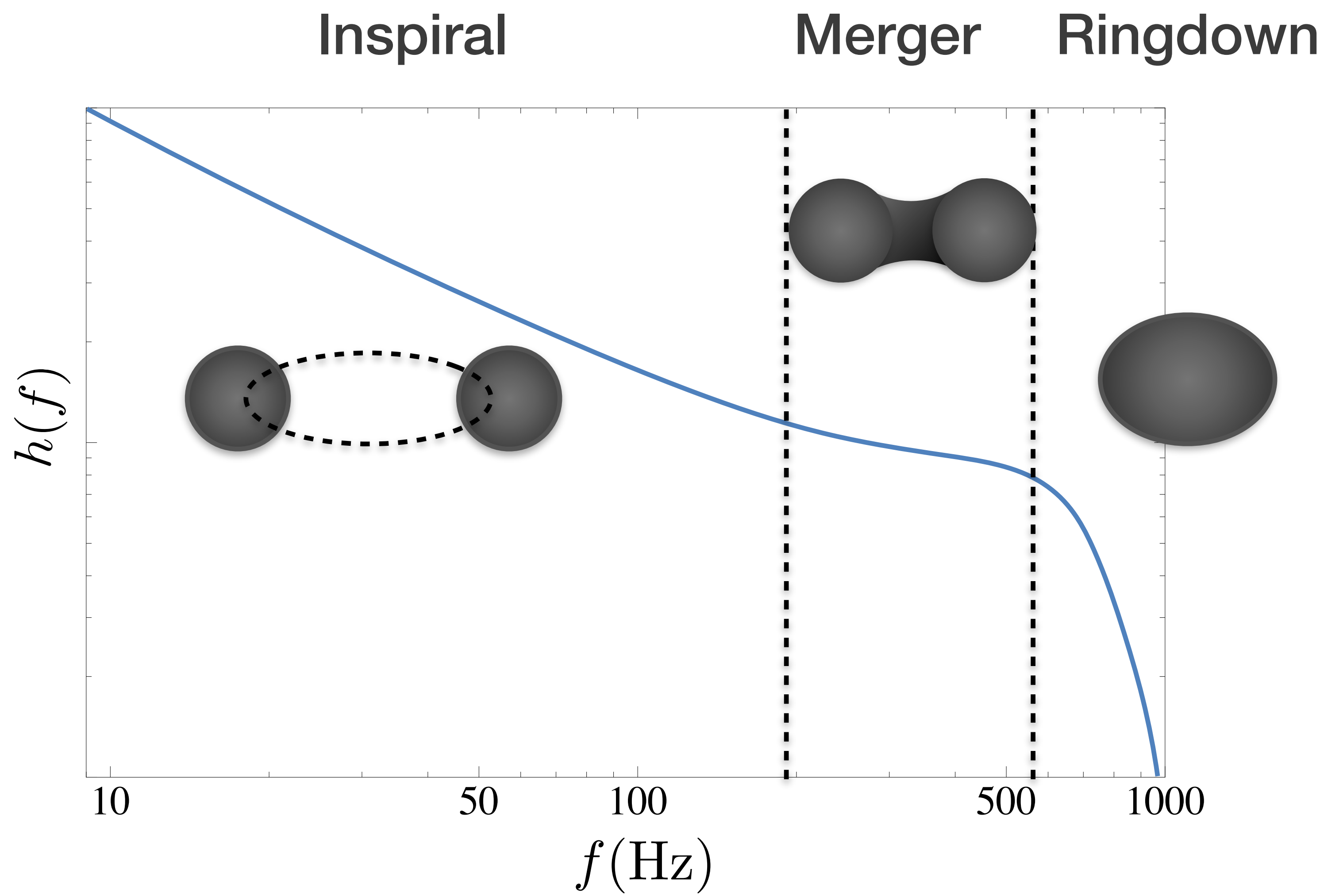
RINGDOWN AND BINARY BLACK HOLES

GWs from compact binaries

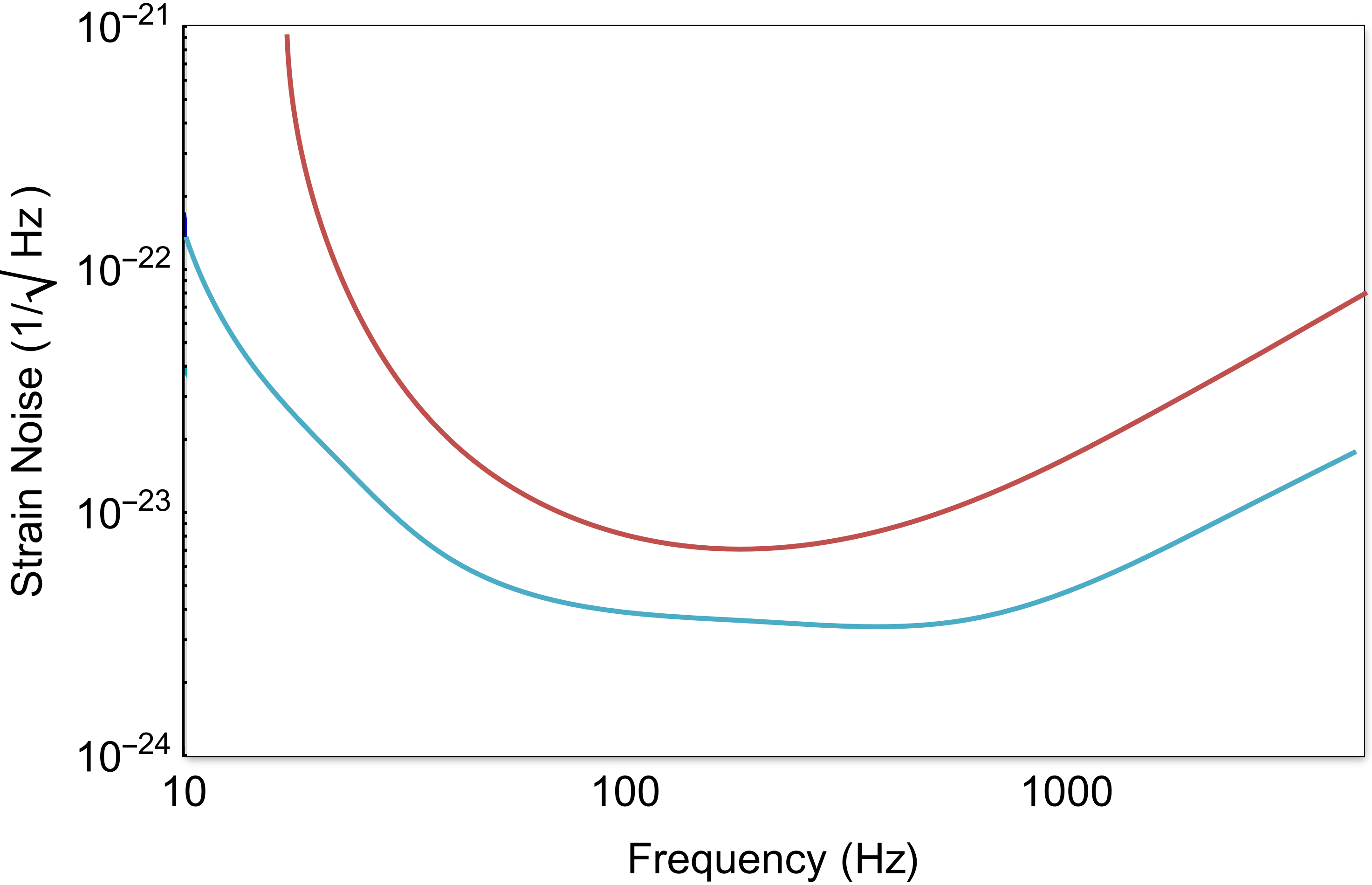


- Entire inspiral-merger-ringdown modeled and calibrated to simulations

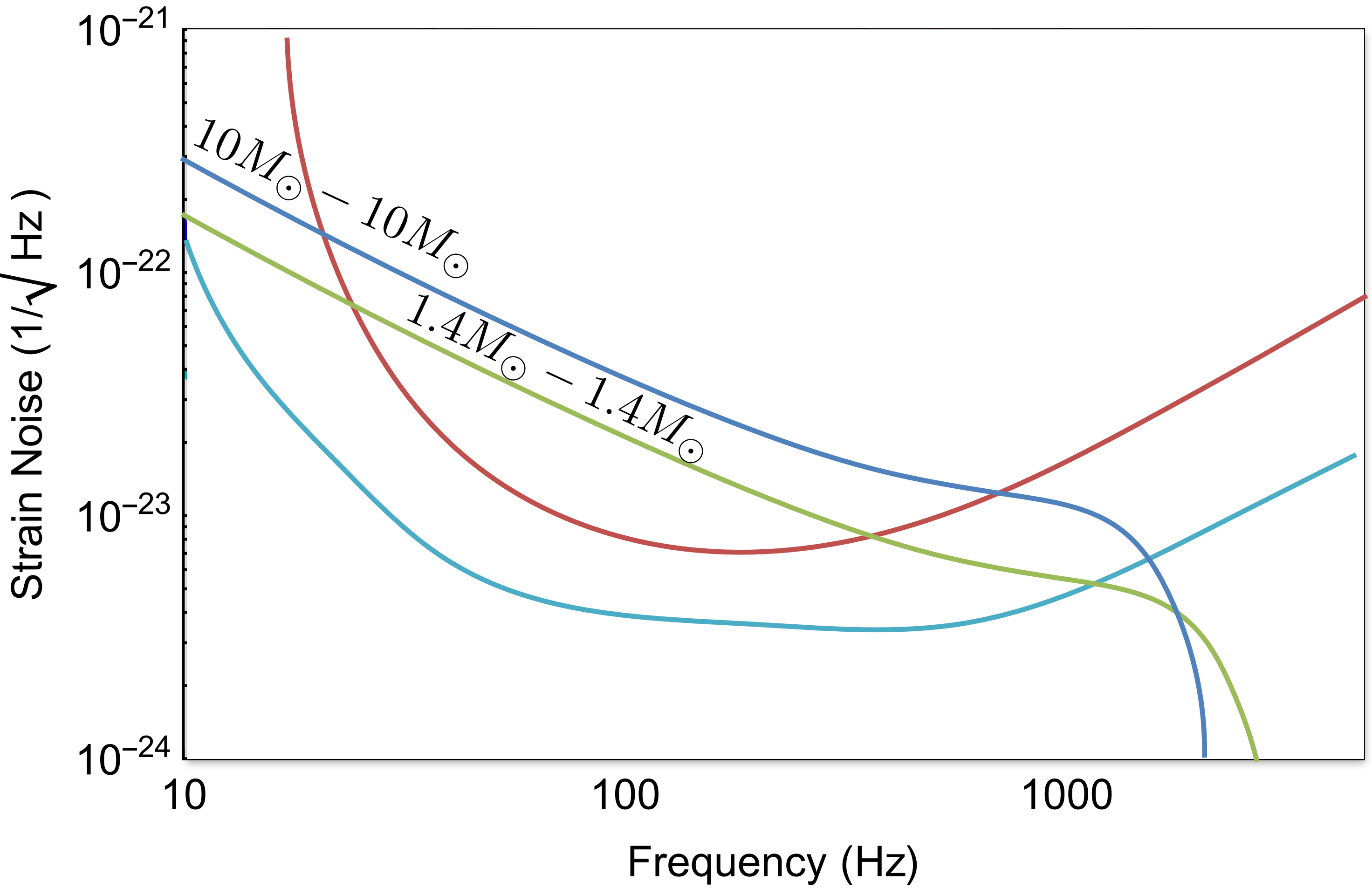
GWs from compact binaries



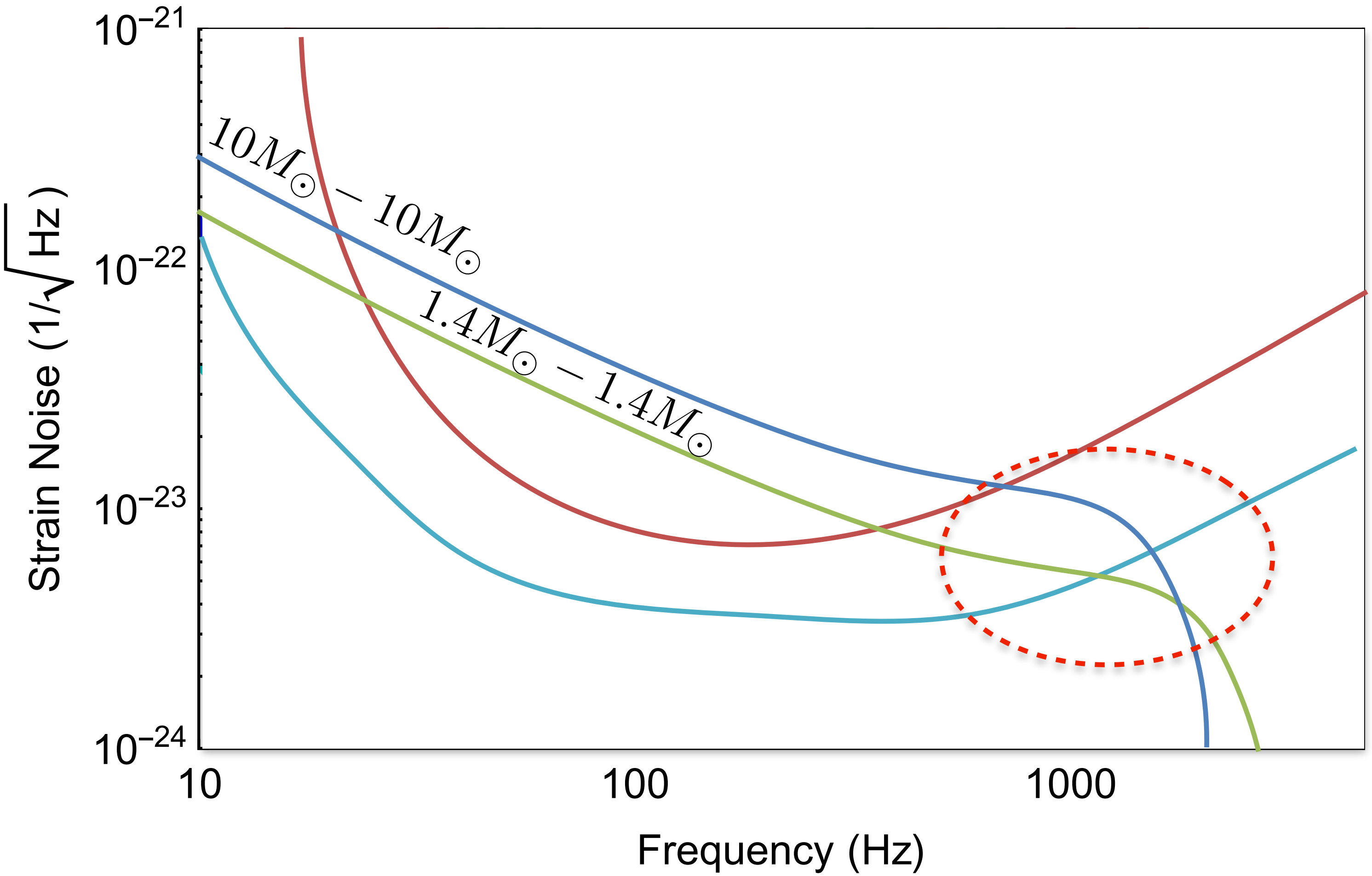
First detection



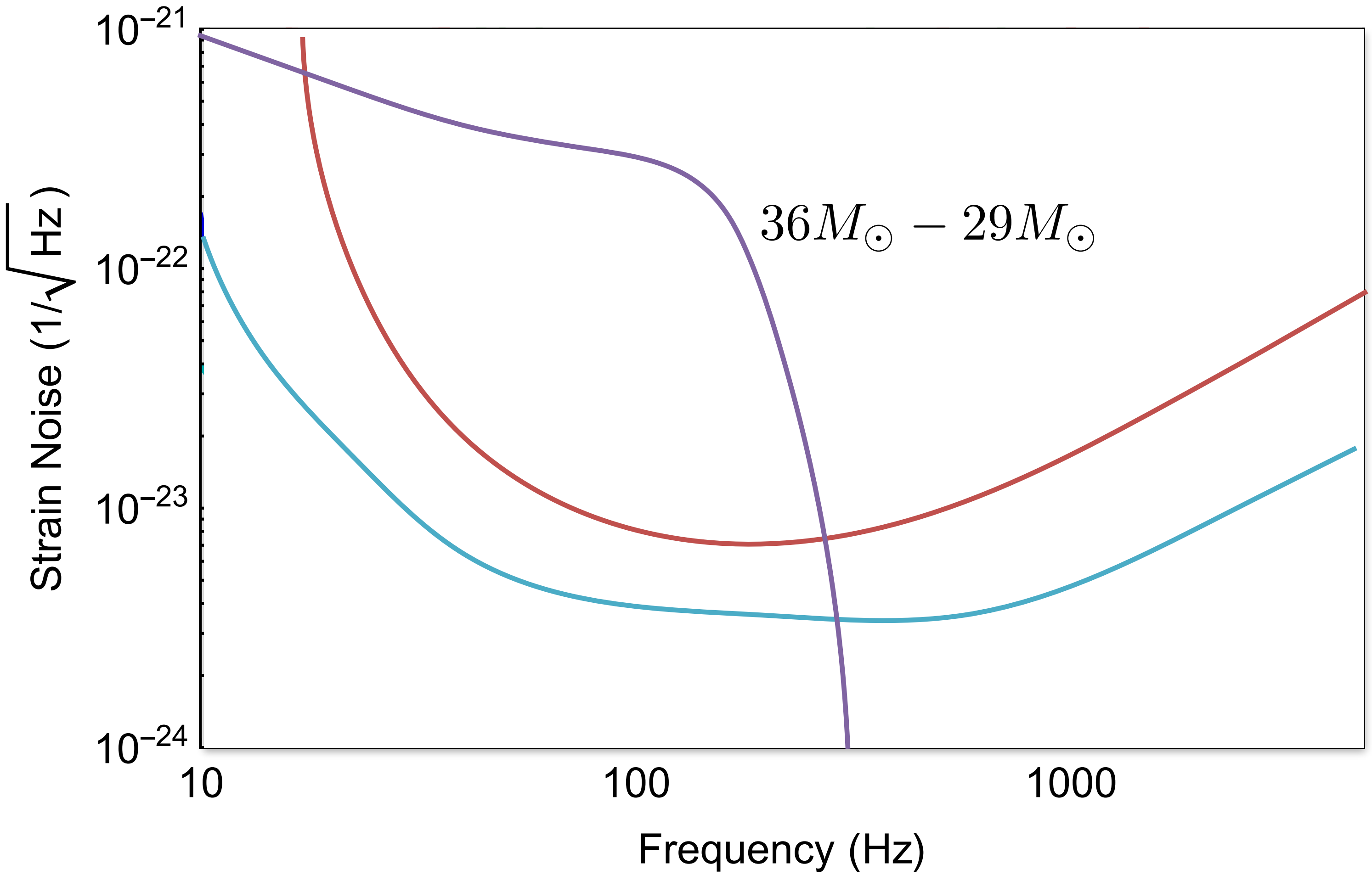
First detection



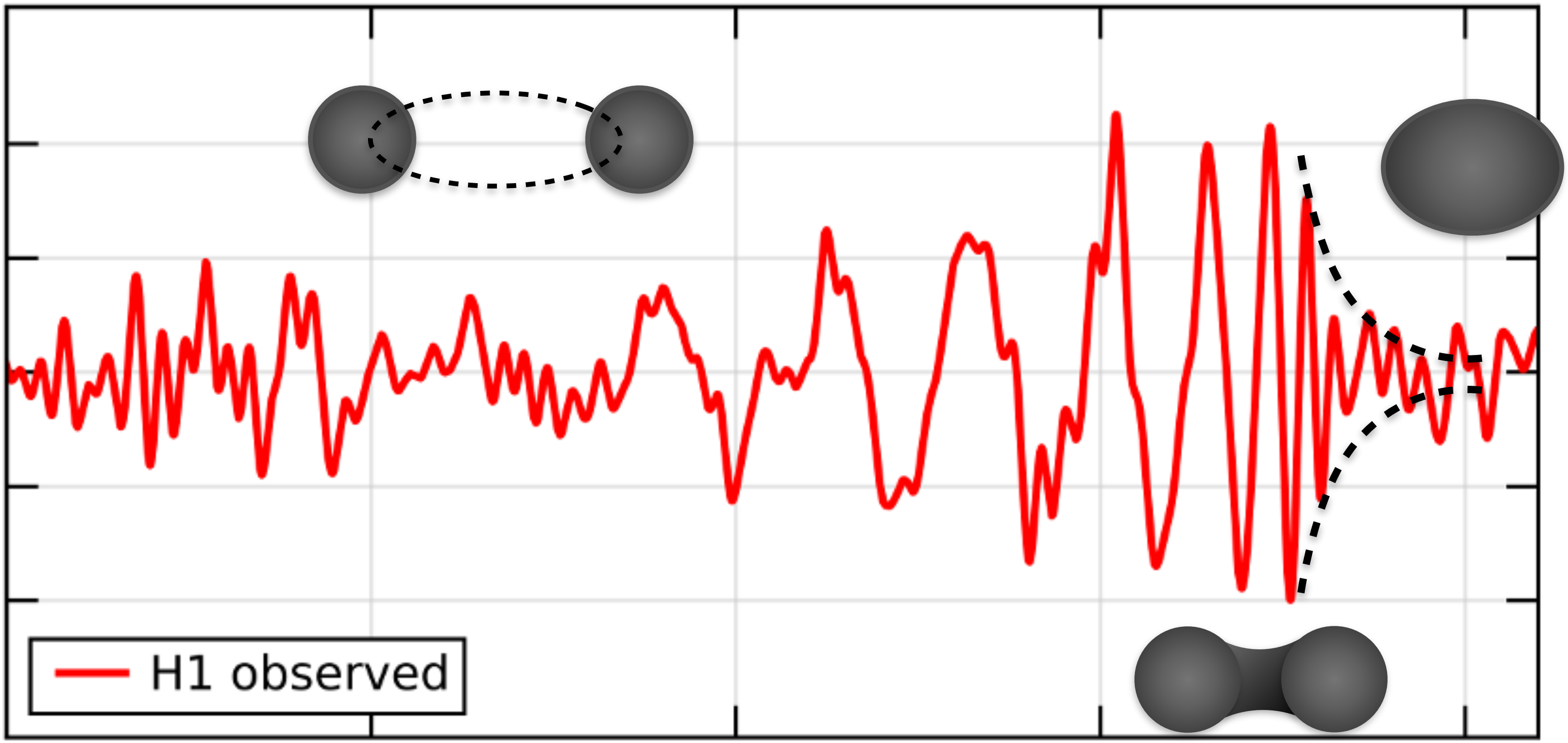
First detection



First detection

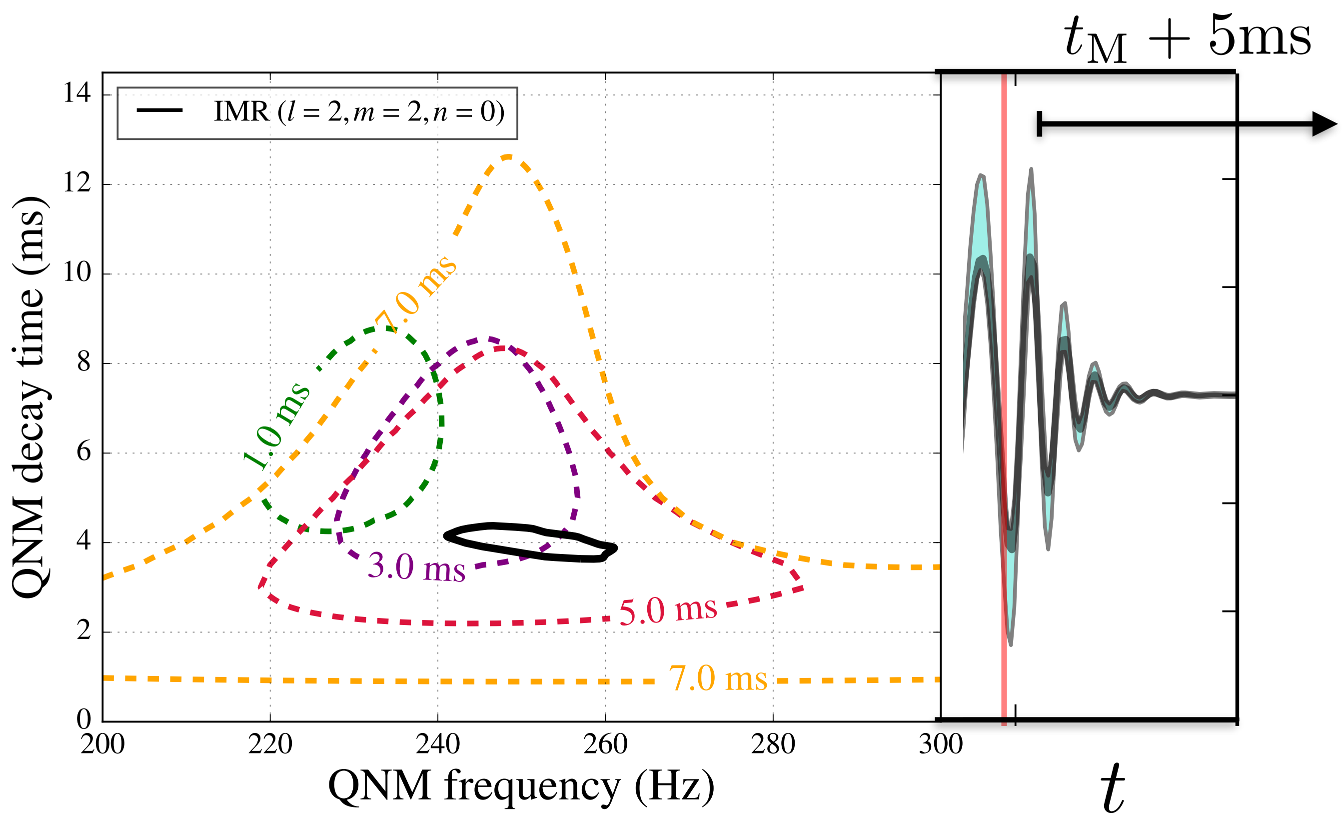


Ringdown of GW150914?



LVC (2016)

The ringdown of GW150914

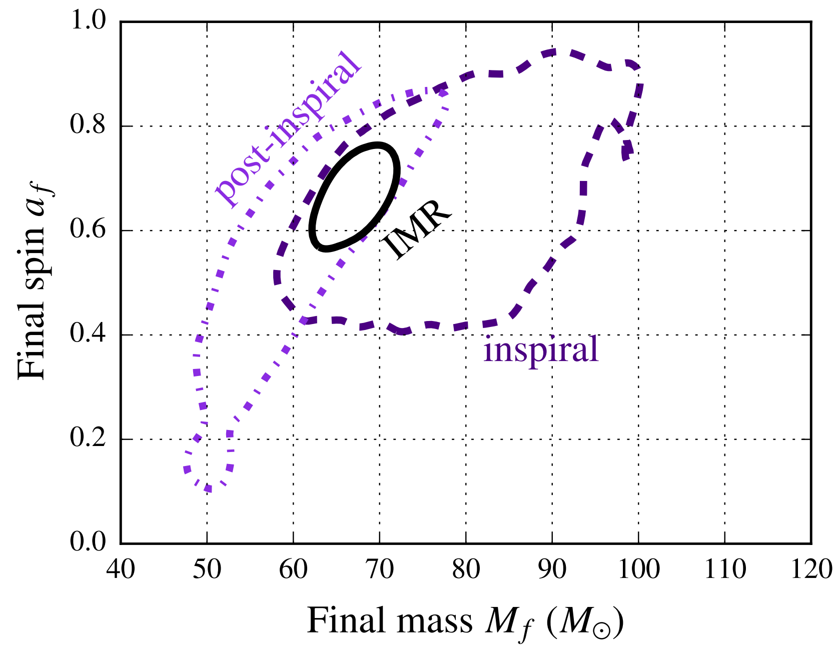


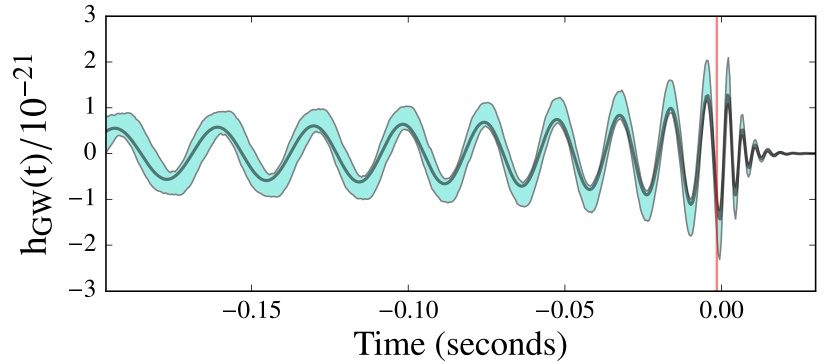
- Freq and decay of lowest overtone for $\ell = 2,\ m = 2$
- Consistent with GR $f \propto 30 \left(\frac{M_\odot}{M} \right) \mathrm{kHz}$

LVC arXiv:1602.03841

Consistency tests of GR

- GW150914: Only 1 mode measured
- Consistency test still possible: split signal
- Compare inspiral with merger-ringdown
- Both consistent with IMR analysis

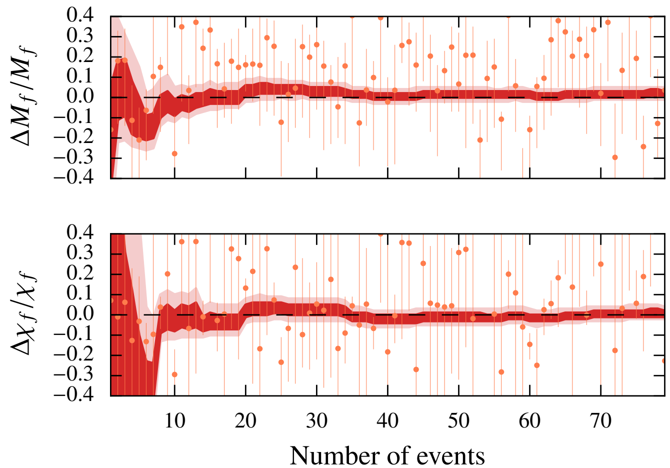


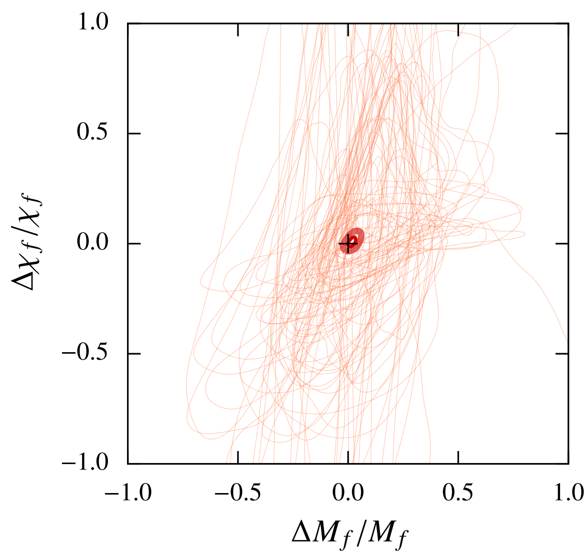


LSC, arXiv:1602.03841

Consistency tests of GR

- Consistency tests can be stacked over many observations
- Ghosh et al. (2016): after ~100 observations at SNR 25, percent level test are achievable





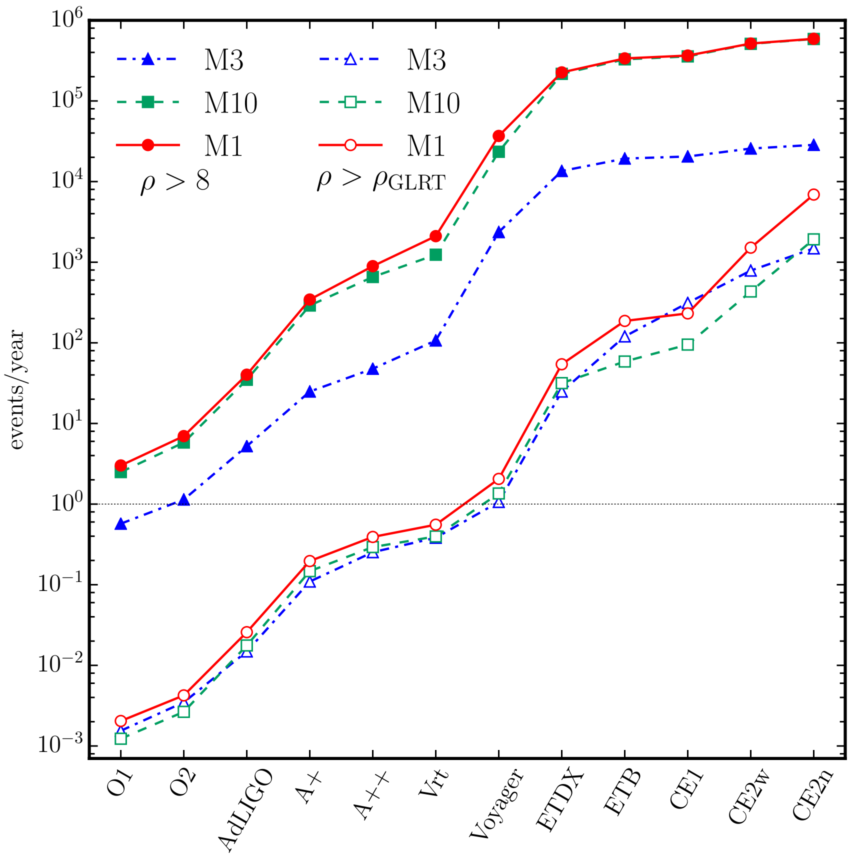
Ghosh et al arXiv:1602.02453

Testing GR with ringdown

- The (2,2) mode dominates
- Large SNR is needed to detect additional modes (~100)

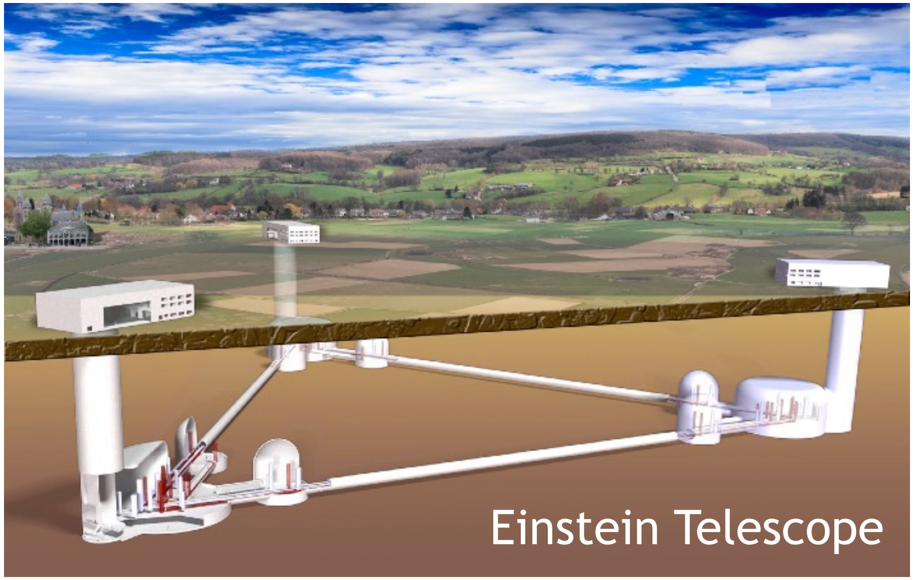
$$\rho_{\rm RD} \propto \frac{M_z^{3/2}}{S_n}$$

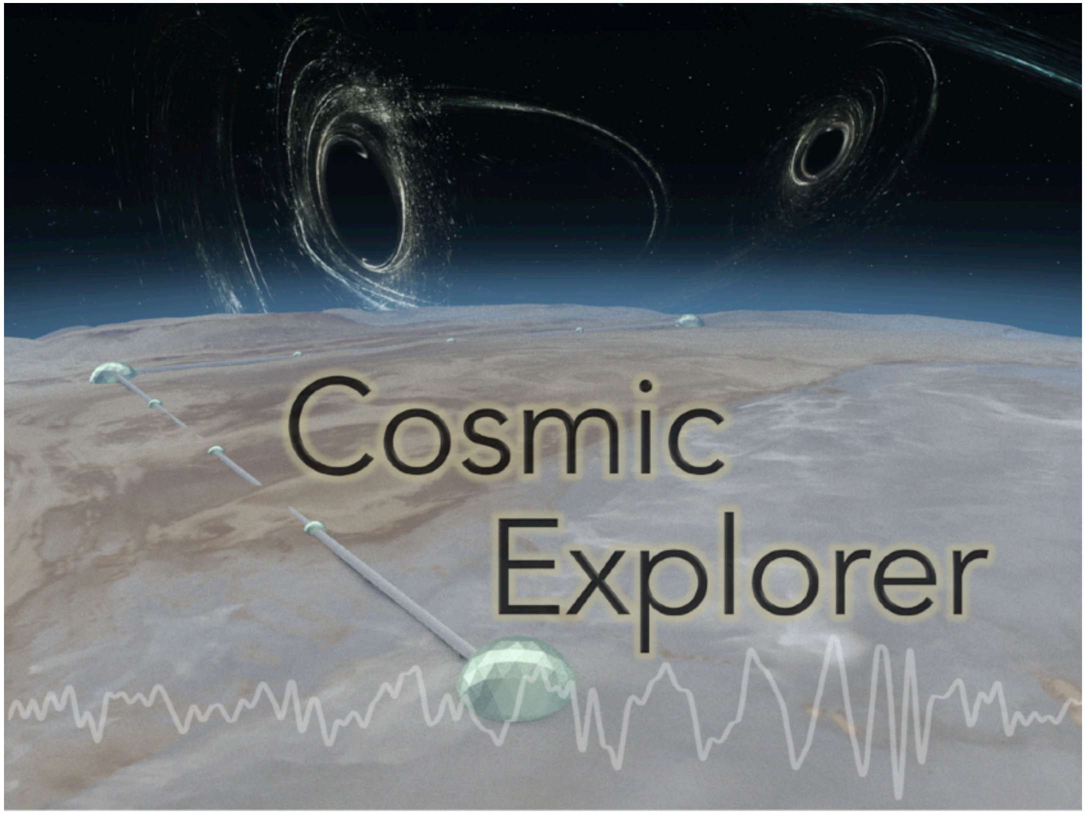
- Hard for ground-based detectors
- Easier for more massive binaries



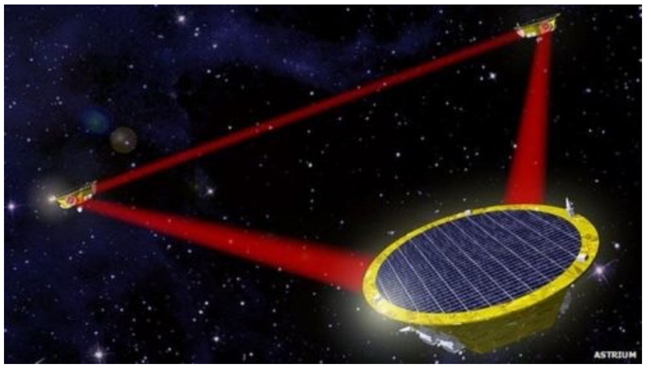
Berti et al. arXiv:1605.09286

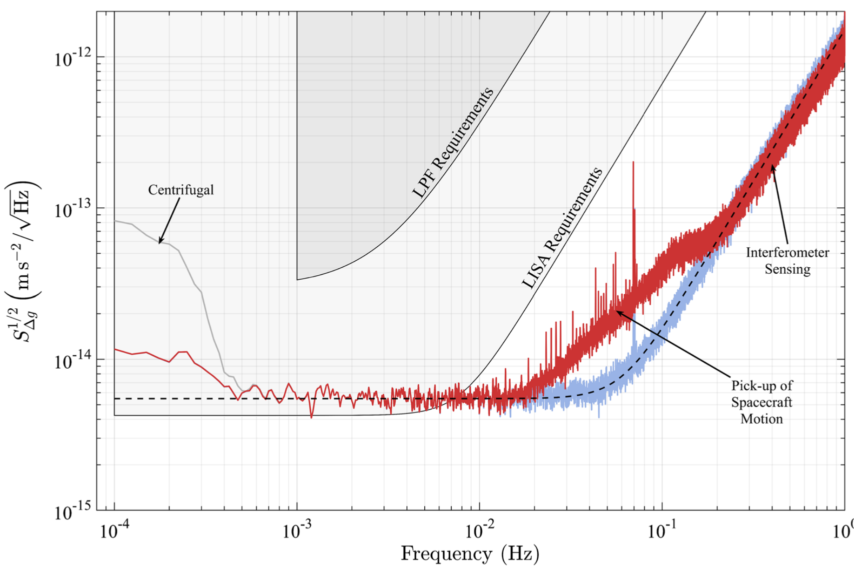
Next Generation Detectors





Space-based detectors



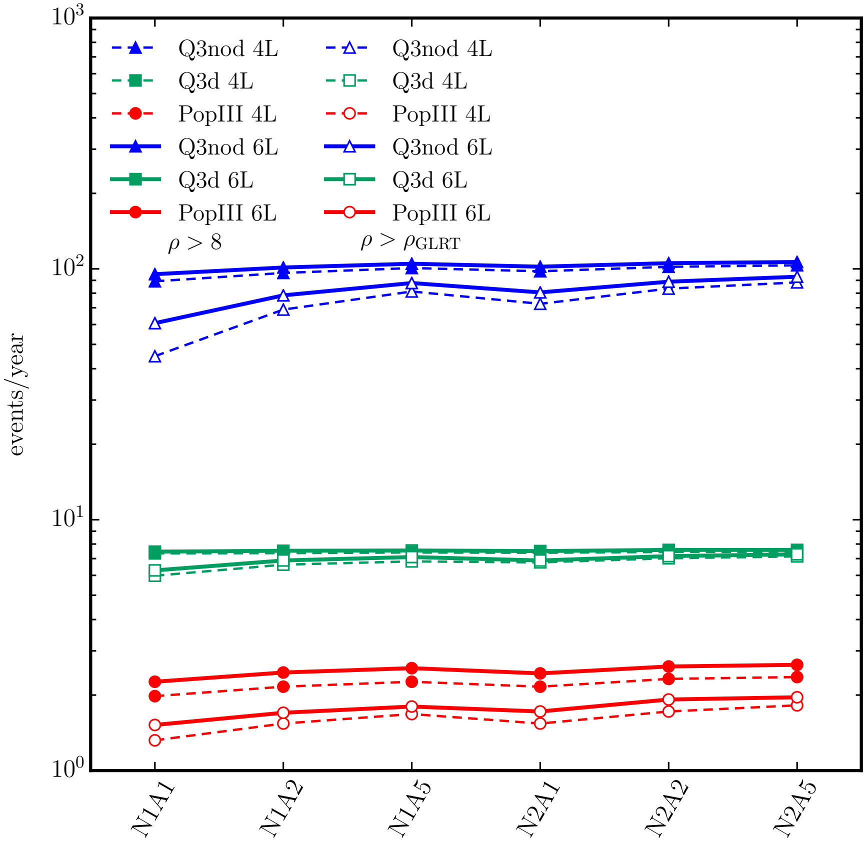


Armano et al. (2016)

- LISA to fly 2034
- Pathfinder a great success
- Space-based missions open many frontiers: SMBH binaries, WD binaries
- Precision tests of GR with EMRIs

Spectroscopy from space

- Space-based BBH detections have huge SNR, $\gtrsim 10^6$
- Ringdown loud enough to measure second mode well
- Many mergers at high redshift
- EMRIs: ringdown at high spin possible



Berti et al. arXiv:1605.09286

Summary

- QNMs outcome of wave eqns on BH spacetimes
- Explored using scalar wave eqs
- QNMs are decaying resonances, for BHs correspondence with null orbits
- QNM spectrum determined by mass and spin
- Rapidly rotating BHs have collective modes (didn’t get to these slides!)