Self-interacting complex scalar field as dark matter

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Abstract. We study the viability of a complex scalar field $\chi$ with self-interacting potential $V = m_0^2/2|\chi|^2 + h|\chi|^4$ as dark matter. Due to the self interaction, the scalar field forms a Bose-Einstein condensate at early times that represents dark matter. The self interaction is also responsible of quantum corrections to the scalar field mass that naturally give the dark matter domination at late times without any fine tuning on the energy density of the scalar field at early times. Finally the properties of the spherically symmetric dark matter halos are also discussed.

Keywords: Dark Matter, Relativistic Bose-Einstein condensates

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Introduction

The evidence for dark matter existence comes from many cosmological and astrophysical observations, e.g. cosmic microwave background (CMB) temperature anisotropy [1], large scale structures of the universe [2] and measurements of galaxy rotation curves [3]. Many models that aim to explain the nature of dark matter, e.g. weakly interacting massive particles (WIMPS), axions [4] and modified versions of general relativity [5]. In alternative one can consider a scalar field as dark matter candidate [6] and in particular a scalar field in a Bose-Einstein condensate (BEC) phase [7]. In [8] dark matter is described through a complex self-interacting scalar field $\chi$ that forms a BEC at early times just after reheating. The scalar field has a renormalizable self-interacting potential $v(\chi, \bar{\chi}) = m_0^2 |\chi|^2/2 + h |\chi|^4$. The self interaction allows the formation of a $\chi$ particle condensate at early times even though the scalar field is not coupled with the standard model (SM) particles [10]. Therefore the scalar field is composed of the BEC in equilibrium with a thermalized particles-antiparticles fluid at temperature $T_\chi$.

The self-interaction also gives thermal corrections to $m_0^2$ that become important at high temperatures $T_\chi \gg T_{1\chi} \gg m_0^2$, where $T_{1\chi}$ depends on the model parameters, giving an effective $\chi$ mass $m_\chi \sim T_\chi$ and therefore a condensate energy density $\rho_\chi^{\text{c}} \sim T_\chi^4$ scaling like radiation. Therefore, at high temperatures both the condensate energy density $\rho_\chi^{\text{c}}$ and that of the thermalized $\chi$ and $\bar{\chi}$ particles $\rho_\chi^{\text{th}}$ scale like radiation $\rho_\chi^{\text{c}} \sim \rho_\chi^{\text{th}} \sim T_\chi^4$ and the condensate cannot dominate over radiation. Finally, at low temperatures $T_\chi \ll T_{1\chi}$ the mass corrections becomes subdominant, the scalar field mass is $m_0$ and the BEC energy density $\rho_\chi^{\text{c}}$ scales as matter so the BEC starts to dominate over radiation. One
can fix the parameters of the model in such a way that the condensate gives the expected energy density \( \rho_{\chi}^c = \rho_{\chi}^{DM} \simeq 0.323 \, eV^4 \) at radiation-matter equality time without any fine tuning on the condensate energy density at early times. Finally we consider big bang nucleosynthesis (BBN) bounds on the extra relativistic degrees of freedom in order to constrain the model. Spherically symmetric solutions of the Einstein system in the newtonian approximation are also considered in order to have a lower limit \( L_H \) for the size of DM halos. A scalar field with a mass \( m^2 \simeq 1 - 10^{-2} \, eV \) and self coupling \( h \simeq 10^{-4} - 10^{-12} \) gives the correct cosmological evolution. The scalar field mass should be compared with the case of scalar field models with no self-interactions, for which one has an extremely low mass of about \( 10^{-23} \, eV \) [7]. Structure formation in this dark matter model have been studied qualitatively in [9].

### BOSE-EINSTEIN CONDENSATE

We consider a scalar field with Lagrangian \( L = \frac{1}{2} \partial_{\mu} \chi \partial^{\mu} \chi - \frac{1}{2} m^2 \chi |\chi|^2 - h |\chi|^4 \) with \( h \ll 1 \). The energy, number and charge density of the complex \( \chi \) field are defined in terms of the phase space distributions \( f_{\chi} \) and \( f_{\bar{\chi}} \) of the \( \chi \) particles and \( \bar{\chi} \) antiparticles. The equilibrium configuration in presence of a BEC is

\[
f_{\chi}(p) = f_{\chi}^{BE}(p) + (2\pi)^3 Q_{\chi} \delta^3(p), \quad f_{\bar{\chi}}(p) = f_{\bar{\chi}}^{BE}(p)
\]

where \( f_{\chi}^{BE}(p) = [e^{\beta(E-\mu)} - 1]^{-1} \) and \( f_{\bar{\chi}}^{BE}(p) = [e^{\beta(E+\mu)} - 1]^{-1} \), \( \beta = 1/T_{X} \), \( \mu \) is the chemical potential and \( Q_{\chi} \) is the number density of the \( \chi \) particles of the BEC. The scalar field is composed of two phases, the BEC and the thermalized gas of \( \chi \cdot \bar{\chi} \) particles at temperature \( T_{X} \). The high temperature thermal correction to the \( \chi \) mass is given by \( (m_{th}^{\chi})^2 \simeq 2h(2\pi)^{-3} \int d^3p (f_{\chi}(p) + f_{\bar{\chi}}(p))/E \simeq (2hQ_{\chi} + hT_{X}^2/3)/m_{th}^{\chi} \), that gives \( m_{th}^{\chi} \simeq \alpha T_{X} \) with \( \alpha \equiv [h(2Q_{\chi}/T_{X}^3 + 1/3)]^{1/3} \). Therefore the effective mass \( m^{\chi} \) of the \( \chi \) and \( \bar{\chi} \) particles will be

\[
m^{\chi} \simeq m_{0}^{\chi} \quad \text{for} \quad T_{X} \leq \frac{m_{0}^{\chi}}{\alpha}, \quad m^{\chi} \simeq m_{th}^{\chi}(Q_{\chi}, T_{X}, h) \quad \text{for} \quad T_{X} \gg \frac{m_{0}^{\chi}}{\alpha}
\]

The number density of \( \chi \) particles is \( n^{\chi} = Q_{\chi} + n_{th}^{\chi} \), where \( n_{th}^{\chi} = (2\pi)^{-3} \int d^3p f_{\chi}^{BE}(p) \), the number density of \( \bar{\chi} \) particles is \( n^{\bar{\chi}} = n_{th}^{\bar{\chi}} \equiv (2\pi)^{-3} \int d^3p f_{\bar{\chi}}^{BE}(p) \), while the energy density of the \( \chi \) field is \( \rho^{\chi} = m^{\chi} Q_{\chi} + \rho_{th}^{\chi} \), where \( \rho_{th}^{\chi} = (2\pi)^{-3} \int d^3p E_{\chi}(p) [f_{\chi}^{BE}(p) + f_{\bar{\chi}}^{BE}(p)] \). Also the charge density is \( Q^{\chi} = Q_{\chi} + Q_{th}^{\chi} \), where \( Q_{th}^{\chi} = n_{th}^{\chi} - n_{th}^{\bar{\chi}} = (2\pi)^{-3} \int d^3p [f_{\chi}^{BE}(p) - f_{\bar{\chi}}^{BE}(p)] \). At high temperatures \( T_{X} \gg m^{\chi} \geq \mu > 0 \) one recovers the usual results \( n^{\chi}_{th} = n_{th}^{\chi} = \zeta(3)T_{X}^{3}/\pi^{2}, \rho_{th}^{\chi} = \pi^{2}T_{X}^{4}/15, Q_{th}^{\chi} = \mu T_{X}^{2}/3 \), see [11]. For simplicity we also define the condensate contribution to the number, charge and energy density as \( n_{c}^{\chi} \equiv Q_{c}^{\chi} \equiv Q_{c}, \rho_{c}^{\chi} \equiv m_{\chi} Q_{c} \).
COSMOLOGICAL EVOLUTION

In the model that we are presenting we suppose that the scalar field $\chi$ is produced at reheating via the inflaton decay. The $\chi$ and $\bar{\chi}$ particles are produced with a charge asymmetry $Q^2 > 0$ via an Affleck-Dine mechanism [12], and then, due to self-interactions, they form a $\chi$-particle condensate. The conditions under which the condensate is formed are studied in ref.[10] where it is found that the condensate forms if

$$R \equiv Q^2/\rho^3_{\chi} = (Q_c/T^3_{\chi} + \mu/3T_{\chi})/(\alpha Q_c/T^3_{\chi} + \pi^2/15)^{3/4} > 0.2\sqrt{h}$$

(3)

Therefore, any realistic choice of the model parameters should fulfill Eq.(3) for any $T_{\chi} \gg T_{1\chi}$. We stress that the presence of the self-interaction is fundamental in this model since it explains how the BEC forms. Now we can study the cosmological evolution of the scalar field $\chi$, see [8]. We stress that the scalar field is composed of two phases which evolve differently, i.e. the BEC with energy density $\rho^\chi$ and the thermalized gas of $\chi$-$\bar{\chi}$ particles at temperature $T_{\chi}$ with energy density $\rho^\chi_{th}$. Since in general $T_{\chi} \neq T_{\gamma}$, where $T_{\gamma}$ is the photons temperature, we define the parameter $k \equiv T_{\chi}/T_{\gamma}$. We also define the following temperatures $T_{1\chi} \equiv m^2_0/\alpha$, $T_{2\chi} \equiv m^2_0$, $T_{1\gamma} \equiv T_{2\gamma} \equiv T_{2\chi}/k$, with $T_{1\chi} \gg T_{2\chi}$ and $T_{1\gamma} \gg T_{2\gamma}$, since $\alpha \ll 1$. Finally we define $t_1$ as the time when $T_{\chi} = T_{1\chi}$ and $t_2$ as the time when $T_{\chi} = T_{2\chi}$ and $T_{\gamma} = T_{2\gamma}$. There are three important epochs in which the $\chi$ field behave differently: at early times $t \ll t_1$, when $T_{\chi} \gg T_{1\chi}$ the $\chi$ mass is dominated by thermal corrections so $m^\chi \approx m^\chi_{th} \approx T^4_{\chi}$. This implies that the energy density of the condensate evolves as radiation since $\rho^\chi_{th} \approx T^4_{\chi}$. Of course, since $T_{\chi} \gg m^\chi$, one has $\rho^\chi_{th} \sim T^4_{\chi}$ and the whole $\chi$ field evolves as radiation with $\rho^\chi \approx \rho^\chi_{th} \sim T^4_{\chi}$. This fact allows to avoid fine tuning on $\rho^\chi$ at early times; At temperatures $T_{1\chi} \gg T_{\chi} \gg T_{2\chi}$ one has $m^2_0 \gg m^2_{th}$, therefore the mass of the $\chi$ particles is simply $m^2_0$ and the BEC evolves as matter with $\rho^\chi \approx m^2_0 Q_c \sim T^3_{\chi}$. Moreover one still have $T_{\chi} \gg m^\chi$, then $\rho^\chi_{th} \sim T^4_{\chi}$ still evolves as radiation. Therefore at the temperature $T_{1\chi}$ the condensate passes from a radiation-like to a matter-like evolution while the thermalized $\chi$-$\bar{\chi}$ particles still evolve as radiation, then the BEC begins to dominate over radiation; At temperatures $T_{\chi} < T_{2\chi}$ below $m^2_0$, also the thermalized $\chi$ and $\bar{\chi}$ particles begins to evolve as matter so the total energy density of the scalar field evolves as matter with $\rho^\chi \sim a^{-3}$. To summarize, the thermalized gas of $\chi$-$\bar{\chi}$ particles becomes non-relativistic at temperatures below $m^2_0$ as usual, but the condensate still evolves as matter at temperatures $T_{1\chi} \gg T_{\chi} \gg T_{2\chi}$ well above $m^2_0$. This last feature is typical of this model and it is due to the fact that thermal corrections to the mass are important only at high temperatures above $T_{1\chi}$. Of course the thermal corrections to $m^\chi_0$ are due to the presence of the $h |\chi|^4$ self-coupling. If self-interactions are turned off, there are no thermal corrections to $m^\chi_0$, therefore the condensate always evolves as matter and this implies a severe fine tuning on its energy density at early times. Moreover one should explain why it starts to dominate just at radiation-matter equality. Since the condensate evolves as a relativistic fluid at high temperatures, it cannot dominate over radiation at temperatures $T_{\chi} > T_{1\chi}$. Therefore one can choose $h$ and $m^\chi_0$ properly in order to ensure a dark matter domination at $T_{\gamma} \approx 0.698$ eV. This helps to explain the cosmological
coincidence problem without any fine tuning on $\rho^X$ at early times.

**CONSTRAINS ON THE MODEL**

In order to constrain the parameters of the model we use the BBN bounds on extra relativistic degrees of freedom parameterized as effective number of extra neutrinos $\Delta_{\nu}^{\text{eff}}$. The contribution of the thermalized $\chi-\bar{\chi}$ particles to $\Delta_{\nu}^{\text{eff}}$ at temperatures $T_{\chi} > T_{2\chi}$ is $\Delta^{\text{th}}_{\nu} = 16/7(T_{\chi}/T_{\gamma})^4$ and imposing BBN bounds $\Delta_{\nu}^{\text{eff}} = 0.054^{+1.4}_{-1.2}$ [13] one has $T_{\chi}/T_{\gamma} \equiv k \leq 0.8$. In the range of temperatures $T_{\chi} \gg T_{1\chi}$ the also the BEC evolves as radiation and one has $\Delta_{\nu}^{\text{eff}}$ is $\Delta_{\nu}^{\text{r}} = 240(T_{\chi}/T_{\gamma})^4 \alpha_{Qc}/7\pi^2 T_{\chi}^3$. The study of spherically symmetric equilibrium configurations of the scalar field in Newtonian approximation shows that the size of DM halos is expected to be greater than a typical length $L_H \equiv \sqrt{h} M_p/m_{\chi}^2$, where $M_p$ is the Planck mass. Therefore, since $L_H$ is a lower bound for the typical size of dark matter halos and therefore any $L_H \leq 100 \text{Kpc}$ is in agreement with astrophysical observations. One can parameterize $\Delta_{\nu}^{\text{eff}}$ as a function of $L_H$ as

$$\Delta_{\nu}^{\text{r}} \simeq 3.34 \left( \frac{L_H}{M_{\text{pc}}} \right)^{2/3}$$

and this relation shows how it is possible to lower the value of $\Delta_{\nu}^{\text{eff}}$ diminishing $L_H$. Therefore an $L_H \leq 0.17 \text{Kpc}$ is within BBN bounds. Choosing $L_H \simeq 0.17 \text{Kpc}$ gives an $m_{\chi}^X \simeq 8h^{1/4} \text{eV}$, that gives a range $m_{\chi}^X \sim 1 - 10^{-2} \text{eV}$ for $h \sim 10^{-4} - 10^{-12}$ [8]. This values should be compared with the case of a scalar field with no self-interactions that requires an extremely small mass $m_{\chi}^X \sim 10^{-22} \text{eV}$ [7].

**CONCLUSIONS**

We have shown that a BEC composed of a complex self-interacting scalar field gives a viable description of dark matter at cosmological and astrophysical level. The scalar field is produced at reheating via the inflaton decay and, due to the self-interaction, a BEC condensate forms. Then, due to thermal correction to the scalar field mass, that in turn are due to the self-interaction, the BEC evolves as radiation with $\rho_{\chi}^X \sim a^{-4}$ and this fact avoid a fine tuning on the scalar field energy density at early times. Finally, at late time thermal corrections to $m_{\chi}^X$ are negligible so the BEC starts to evolve as matter and begins to dominate over radiation and dark matter domination begins. The model is viable for a broad range of parameters and in particular a value of $L_H \simeq 0.17 \text{Kpc}$ gives a $\Delta_{\nu}^{\text{eff}}$ within the BBN bounds and a mass $m_{\chi}^X \sim 1 - 10^{-2} \text{eV}$ for $h \sim 10^{-4} - 10^{-12}$.

**REFERENCES**