An introduction to general relativity, black holes and gravitational waves

Nora Bretón

Dpto. de Física, Cinvestav-IPN, Apdo. 14-740, D.F., México.

Abstract. We introduce, at a basic level, the concepts that are necessary to get acquainted with two of the most interesting fields of research which emerged from classical general relativity: black holes and gravitational waves.

Keywords: exact solutions, gravitational waves, extradimensions
PACS: 04.30.-w, 04.50.Gh, 04.20.Jb, 11.10.Kk

INTRODUCTION

It was 1915 when A. Einstein presented his new theory of gravitation to the Prusian Academy of Sciences. At that time nobody suspected the huge legacy and amount of research directions that, 100 years later, would have arisen from it: from understanding what the universe is made of, its origin and final destination (cosmology); why, in spite of being homogeneous and isotropic at large scales (300 Megaparsecs), at smaller scales it has a structure of galaxies, galaxy clusters, etc.; the nature of highly dense bodies and the way in which the spacetime is affected when violent astrophysical events, like the colision of black holes, occur, producing so-called gravitational waves.

The seed for general relativity theory was special relativity theory. And the special ingredient to formulate special relativity was Albert Einstein's deep physical insight, having as his most relevant assumption that, regardless of the reference frame we formulate the laws of physics in, such laws should describe the same phenomenon in the exact same way.

RELATIVITY OF SPACETIME

1905 has been called the miraculus year in A. Einstein life due to the enormous contributions that came along that year: he published four works that completely changed the view of physics in very different branches. He proposed an explanation for the Brownian motion; in his Ph. D thesis, he gave an approximate size for the still unaccepted molecules. In the paper *On a heuristic point of view concerning the production and transformation of light* he proposed that light interacts with matter in a quantized way, and then in one of the sections he explained how the photoelectric effect works, which made him win the Nobel Prize in 1921. Besides he also solved an old controversy that has to do with the distinct coordinate transformations that were appropriate to classical mechanics (Galilean transformations) and to the laws of electrodynamics, set as the

Maxwell equations (Lorentz transformations). To carry out his hypothesis of universal validity of the Lorentz transformations he gave up the, popular at the time, concept of *aether*. This is known as special relativity theory, which he published in the Analen der Physik entitled as *On the electrodynamics of moving bodies*. In another paper the same year *Does the inertia of a body depend upon its energy content?* he derived the most famous equation in the world: $E = mc^2$.

Turning back to special relativity, it is based in two empirical facts, or principles as Einstein called them:

1. The output of an experiment shall obey the same rules, regardless of the place the experiment takes place in. In other words, all inertial reference frames are equivalent when it comes to derive the laws of physics. Recall that inertial reference frames are the ones that move relative to each other with constant velocity (uniform relative motion).

2. Every inertial observer sees light traveling at constant velocity $c = 300000\text{km}/\text{seg}$, independently of the movement of the source of light and of the reference frame in which this is measured. Moreover this is the maximum attainable velocity, nothing can travel faster than light.

Starting from these two principles, if we now try to check consistency with the usual rule to add velocities, we will find a disagreement with the second principle (that light has the same constant velocity for every observer) as the following thought experiment shows:

A train travels with velocity V, and inside, a passenger walks with velocity w. For someone who watches the scene from outside, standing at the platform, the passenger inside moves at velocity $V + w$ if walking in the same direction than the train or at $V - w$ if the passenger is moving in the opposite direction. Now let us suppose that a light beam is emitted in the platform; the standing observer will measure a beam's speed of c. Which would be the result of measuring the beam's velocity for the passenger in the train?. If we stick to the usual rule for velocity addition, the passenger (if he is standing still in the train) shall measure $c_p = c \pm V$, depending on whether the beam and the train travel in the same (plus) or in opposite directions (minus). This result contradicts the second principle of special relativity theory! Because such principle states that both observers, inside the train and at the platform, must observe a velocity of c for the beam. Therefore, there is something wrong with our rule for summing up velocities, which doesn't agree with experience.

Demanding that every observer measures the same speed of light, regardless of his/her reference frame, leads to several consequences, for instance, the relativeness of the concept of simultaneity. Two events that occur simultaneously in a reference frame are not necessarily simultaneous in another (i.e. one of the events may occur before the other, provided that causality is not violated). This can be explained by means of the following gedanken experiment [1]: Go back to the train and think in a sparkle that produces simultaneously two light beams, one traveling toward the front end of the wagon (point A) and the other in the opposite direction, toward the rear of the wagon (point B, see Fig. 1). Let us compare the experience of a passenger inside the train and someone else outside, standing at the platform. According to the passengers in the train, light beams reach points A (front end of the train) and B (the rear of wagon) simultaneously. But from the viewpoint of the guy at the platform, the light signal reaches point B in less time than point A, because point B is getting closer to the beam as the train moves,

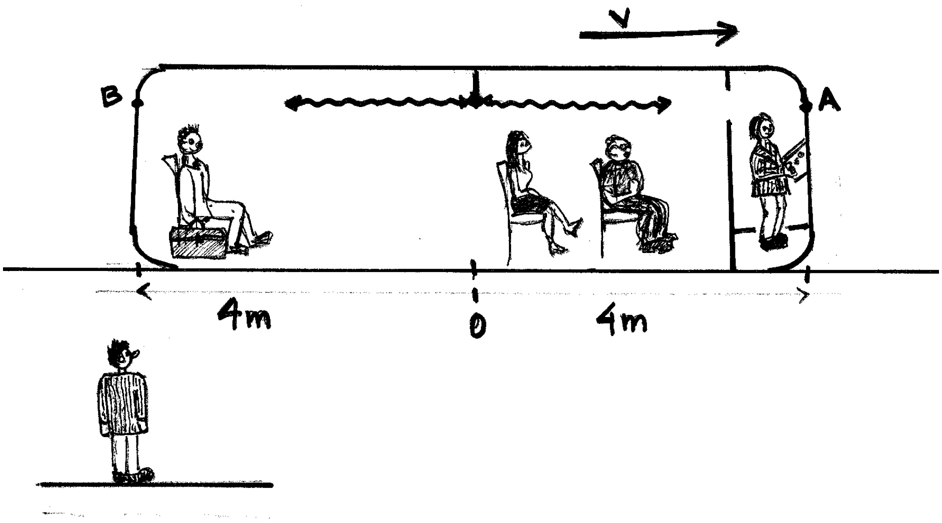

FIGURE 1. Events that are simultaneous in some reference frame, may not be so in another.

while point A is receding. Therefore, events that are simultaneous in some reference frame, may not be so in another.

Other consequences of relativity are effects on the measurement of lenght and time, which depend on the velocity of the observer [2]; these effects are known as *lenght contraction* and *time dilation*. Let us consider two inertial reference frames K and K' in relative motion at constant velocity v. Let be x be a lenght along the X axis as measured by the observer in K; let be x' be the lenght along the X axis as measured by the observer in K'. The lenghts x and x' are not the same, but they are related by means of the Lorentz transformation $x' = x(1-\frac{v^2}{c^2})^{1/2}$. For instance, if the length of a rod, according to K, is $x = 1$m, and the relative velocity between K and K' is $v = 0.5c$, then the observer in K' measures a length of $x' = 86.6$cm for the rod. The maximum lenght is measured in the frame that is at rest. At any other system in movement relative to that frame, the lenght is reduced by an amount of $(1-\frac{v^2}{c^2})^{1/2}$. This effect is known as Lorentz's *lenght contraction*.

Time dilation is a shocking effect related to time measurements. For the frames in relative motion K and K', and for a watch fixed with respect to K', let us compare the measured interval times Δt and $\Delta t'$, respectively. They relate to each other by $\Delta t' = \Delta t(1-\frac{v^2}{c^2})^{1/2}$. This indicates that the time intervals in K last longer than those of K'. For instance, if in K' a time interval of one second elapses, then the very same interval lasts 1.15 seconds in K if the relative velocity between K and K' is $v = 0.5c$. Another way to put it is to say that time goes slower in moving frames. Several paradoxes arise from this effect, like that of the twins that are separated and one of them travels with huge velocity, far away from his brother; years later the first gets back and when they meet again, the traveler looks younger than the guy that stayed at home [3].

These effects are unobserved in our ordinary experience, because to be noticed, huge velocities (close to that of light) should be involved. By huge we mean HUGE: for instance, the launching velocity of a rocket is about 8700Km/hr that is, $v = 0.029c$, which is rather small (about three hundredths of c!). Moreover, $(v/c)^2 = 8.2\times 10^{-4}$, which contracts every meter by 4 tenths of a millimeter. However, in high-energy particle accelerators, velocities comparable to that of light ($0.5c$, $0.7c$) can be reached; and at those velocities, all of the aforementioned relativistic effects are our bread and butter. This effects can also be observed in the earth muons, who travel distances greater than their short mean lifetimes at rest would allow.

Other experiments have been carried out with atomic cesium clocks that travel in airplanes, and their measurements are compared with those of similar clocks that stay on earth. Depending on whether the airplane is traveling toward the east or the west, the results turned out to be clearly different.

Minkowski spacetime and curved spacetime

As we have had the opportunity to appreciate, each reference frame has its own time. In order to determine an event on a reference frame, it does not suffice to have its spatial coordinates, but we still need to know the instant of time the event takes place in. This is, an event is completely described by three spatial and one temporal coordinate: (x,y,z,ct). Our reference frame grew one new dimension, namely time. The space equiped with time is the spacetime; if it is $\Re^3$ plus time, it is called *Minkowski spacetime*.

As we mentioned before, neither lenght nor time intervals are absolute, but they depend on the reference frame of the observer that measures them. However, there are quantities which remain constant and do not depend on the reference frame. These quantities are of extreme importance, since they do not change from one reference system to another. They are called *invariants*. One of these quantities is the four-dimensional lenght or *distance of separation* between two events,

$$\begin{aligned} ds^2 &= (\Delta x)^2+(\Delta y)^2+(\Delta z)^2-c^2(\Delta t)^2 \\ &= (\Delta x')^2+(\Delta y')^2+(\Delta z')^2-c^2(\Delta t')^2 = \text{constant}. \end{aligned} \tag{1}$$

where $\Delta x = x_2 - x_1$, etc., refer to the intervals whose extremepoints are (x_1,y_1,z_1,t_1) and (x_2,y_2,z_2,t_2) in the primeless system, and the analogous for the primed quantities. Notice that there can be null (light beam trajectories), as well as negative, lengths. Thus, this determines a metric that is not positive definite.

Einstein's first thoughts to marry gravitation and spacetime

Newton's law of gravity had reigned for about two centuries: it explained the orbits of planets around the Sun, orbits of satellites around planets, ocean tides and the falling down of objects on Earth. Even when Uranus seemed to violate the gravity law (1781), it was discovered shortly afterward that the anomaly in its orbit was produced by the influence of a so far unseen planet: Neptun (1846). In short, Newton's law of gravity was capable of explaining all the known evidence relating to gravity. However, at the beginning of the twentieth century, the orbit of Mercury was discovered not to be as accurate as Kepler's law commands: Its perihelium showed a shifting resulting in a non-closed orbit (i.e., after one whole turn around the Sun, Mercury does not return to the same position). This flaw gave Einstein the insight that something was wrong with Newton's gravity law. In spite of the lack of solid experimental results, Einstein was guided mainly by his intuition about how the laws of physics should look like (so the fact that Newton's law of gravity depended on a particular reference frame also played a role in Einstein's thoughts about the wrongness of such law).

With those fuzzy ideas, in 1907 Einstein was asked to write a review on his relativity theory (special at that time). While doing it, he realized that gravity was not included in the theory, since it only considered inertial frames, i.e. frames that are free of forces.

Thinking about how to include gravitational force in the relativity theory, while he worked at the patent office in Berna, he was struck by a happy idea, which he would later think of as *the luckiest though of my life*: while falling freely, a person can not feel his/her own weight.

In other words, a constant gravitational field can be mimicked by an accelerating system. This is the *equivalence principle*, equivalence which arises because gravitational mass, the one that exerts gravitational attraction, is the same as inertial mass, the one that opposes resistence or inertia to be moved. This is one of the basic ideas of general relativity.

Basic ideas of general relativity.

The equivalence between inertial and gravitational mass was already noticed by Galileo (1600): all objects fall with the same acceleration, no matter how much mass each have. A stone and a feather left falling down from the same height, will both get to the floor at the same time (neglecting air resistance). This is so because gravitational force is proportional to mass; this does not happen with other forces like, for instance, electrical force.

Many experiments have been done to test this equivalence, beginning with Eötvös in the nineteenth century, till now. The precision reached so far is of about ten digits or more. The hint that this idea gave to Einstein was that the description of gravitational force can be done in a way different than that of Newton's. Instead of vectors, one can equivalently describe the trajectories of bodies (particles) under the influence the gravity. The presence of massive bodies change trajectories: just think of how Earth attracts bodies, making they fall directed to its center. The paths that particles follow are, in general, curved. These curves are determined by the bodies that produce the gravitational field. Moreover, if spacetime is curved, even light trajectories should be bend. In this approach, general relativity is a field theory described through geometry, instead of forces. Those paths or trajectories are not straight in general, therefore, geometry, particularly the study of curves in space was required. Later on this idea was called by Wheeler (1960) as geometrodynamics. So, we say that general relativity is a field theory, where the field is geometry and, as opposed to the Newtonian conception, there is no action at distance.

However, in our neighbourhood we do not notice any curved trajectory. Locally our space seems to be flat like a sheet of paper on a table. Thus, spacetime should locally be the one of special relativity, namely Minkowski spacetime, or Euclidean space at each point and its neighbourhood. Then at each point we should be able to find an inertial reference frame. Moreover, any gravity theory should be consistent with the fact that if Newtonian gravity is so succesful, that is because it is strikingly precise within certain limits (with a broad range of applicability), therefore, the new gravity theory, whichever it be, must be Newton's gravity at certain limit.

Surfaces and curved spaces

Let us think about the curvature of spacetime. In 1854, Bernhard Riemann suggested that the differential geometry of our four-dimensional spaces should be determined by external forces. Riemann, Levi-Civita, Gauss and other mathematicians had already studied curved surfaces. Therefore, in order to formulate the general relativity theory, Einstein had to learn what now we call Riemannian geometry.

Although we are familiar with curved surfaces, we cannot visualize curvature in three or more dimensions, since we cannot immerse our three-dimensional space in a four-dimensional one and see it "from the outside".

As opposed to a plane space where points can be labelled by Cartesian coordinates, and the axes are perpendicular to each other in each point, in Riemannian or curved spaces, the axes are, in general, not perpendicular to each other. Let us consider, for instance, the distance on a sphere of radius a, where (x,y,z) are Cartesian coordinates in space,

$$x = a\sin\theta\cos\phi, \quad y = a\sin\theta\sin\phi, \quad z = a\cos\theta$$

$$ds^2 = dx^2 + dy^2 + dz^2 = a^2 d\theta^2 + a^2\sin^2\theta d\phi^2,$$

>From this we see that, in a Riemannian metric, ds^2 is a quadratic form.

Besides curved spaces, we need a temporal coordinate. This leads us to a generalization to non positive-definite metrics, i.e., spaces where the distance between two points is not necessarily positive. The most known example of this kind of spaces is Minkowski space: $ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$. This is an euclidean space with an additional temporal coordinate, and we say that the *signature* of this space is $(-+++)$. Signature is invariant with respect to real coordinate transformations.

Some gravitational effects

In which precise manner does gravity curve spacetime?

I hope that, by now, you are already convinced that time is relative, that it is not the same to every observer, but rather it depends on his/her movement. Well, time is also affected by gravity: Two observers staying in a building of twenty floors, one at the basement and the other on the last floor, will not measure the same time intervals. In fact, an astonishing result has recently been settled: last winter, a difference between measurements on different heights has been established, with as little difference between heights as 22.6 meters. The difference in frequencies (like tic tacs of a clock) was 4.92×10^{-15}. This result is of great importance in the search for a theory of quantum gravity and could have also practical implications, such as improving the accuracy of global positioning systems. A clock near a massive object runs slower than a clock situated far from the massive source. At the Sun's surface a clock works slower (one part in a million) than a clock far away. A clock on the surface of a neutron star works 70% slower than another clock situated far from the star.

The shift on wavelenghts of light is another effect produced by a massive source: Light wavelenght is shifted towards red (large wavelenghts) when coming from a gravitational field. The mechanical analogue of this effect is when throwing up a stone, it losses kinetic energy while going up, against gravitational field; so the light, traveling away from a gravitational field, losses energy, this loss resulting in the red-shifting of wavelenght (recall that $\nu\lambda = c$).

These effects are derived from Einstein's equations, the field equations that determine in which precise way matter affects the geometry of spacetime.

Einstein's equations

The basic idea of Einstein's theory of gravity is the geometrization of gravitational force: all properties and influence of the gravitational field, are manifested through the curvature of spacetime. Gravity and curvature should be incorporated into mathematical equations with the following requirements:

(i) The equations should be formulated in tensorial languaje, in order to embody the fact that nature is independent of the reference frame.

(ii) Should be partial differential equations of at most second order in the functions to be determined; in this case to determine the metric g_{ij} because

(iii) in the weak field limit, the equations should reduce to Newtonian gravity, i.e. a Poisson equation for the gravitational potential ϕ with the mass density ρ as source.

$\nabla^2\phi = 4\pi G\rho$

(iv) The source of gravitational field, the so-called *energy-momentum tensor* should be the analogue to mass density.

(v) Flat spacetime should correspond to the absence of matter ($\rho = 0$).

Such equations were sought by Einstein for years, like eight years or so. Assisted by his good friend from academic years, Marcel Grossman, he had to learn to use new mathematical tools like Riemannian geometry and differential geometry. In November 1915, at the Prusian Academy of Sciences, Einstein introduced his new theory of gravitation, comprised in the tensorial equation,

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}. \tag{2}$$

To substitute the only Newtonian equation

$F_G = G\frac{Mm}{r^2}$.

Tecnically, general relativity is a field theory with a tensorial potential, while Newtonian gravity theory or Maxwell electromagnetic theory have, respectively, a scalar and vectorial potential, whose derivatives are related to the field itself. Eq. (2) are actually ten equations; corresponding to ten metric components to be determined, $g_{\mu\nu}$, since μ and ν can take four values each (4×4) and considering that $g_{\mu\nu} = g_{\nu\mu}$.

The left-hand side of (2) is related to the geometry of spacetime: $R_{\mu\nu}$ measures the geodesic deviation, while $g_{\mu\nu}$ tells us how to measure distances and time. R measures the spacetime curvature, is analogous to Gaussian curvature κ for surfaces. The right hand side of the equation is the matter content of spacetime, other than gravitational mass; in here you can put electromagnetic fields, fluids, scalar fields (like axions, dilatons, spinorial fields, etc.). Therefore, Einstein's equations express the compromise between spacetime geometry and the existing matter, in such a way that each one influences the other, being the curvature the manifestation of the massive content of that spacetime. The main difference with respect to the old approach is that matter evolves no longer through a static spatial scenario, where all clocks in universe agree in their time measurements, but rather now spacetime is an active actor that affects matter dynamics; and in turn the matter content, through the energy-momentum tensor $T_{\mu\nu}$, determines how the geometry is. Moreover, Einstein's equations are not linear, where by linear effects we understand those that are proportional to the causes, those where small variations in initial conditions lead to small changes in the response, not much different than the former. In nonlinear processes this is not the case.

Sometimes we use the approximation in which some weak fields or particles do not affect the geometry; the structure of space is completely determined by $T_{\mu\nu}$, assuming that the latter in turn does not affect curvature. In this case, the particles or fields are called test particles or test fields, i.e., fields that do not produce in turn new gravitational fields, but rather are only affected by the already existing gravitational field.

Freedom in choosing any reference frame should be preserved, i.e. any coordinate system should be able to be used to describe physics (of course some coordinates are more apropriate to some problems, depending on the geometry of the analized system); therefore we need four degrees of freedom, one for each coordinate. In other words, the physics derived from the metric tensor (Einstein's equations) should not depend on the special coordinate system chosen. Keeping these four degrees of freedom, it turns out that only six of the equations are needed to determine the metric tensor, $g_{\mu\nu}$. Once the metric tensor is known, we then know how to measure distances in that spacetime, and hence we can determine also particle trajectories, etc., through the line element,

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu. \tag{3}$$

Testing the new gravitational theory

Why would one want to use Einstein's equations to describe gravity, if they are way more complicated than the unique Newton's equation, $F = GM_1M_2/r^2$?

We have commented above about light discrepancies observed in Mercury's orbit: its orbit does not close, after one revolution around the Sun Mercury does not get back to the same place. Therefore we would expect the new theory to predict correctly the orbit and this is in fact one of the achievements of Einstein's theory.

There are also other predictions that were confirmed by observations, namely,

1. The precession of the perihelium of the orbits in binary systems, i.e. two compact objects rotating around each other. In our solar system, the effect is noticeable only in Mercury's orbit. Nowadays it is observed in several binary systems.

2. Light rays are bended when passing near a massive object. This effect was confirmed during a solar eclipse, as early as in 1919, by Sir Arthur Eddington.

3. Doppler gravitational effect, consisting in the delay in clocks, or the shift to red of light waves. The effect has been observed in light coming from stars; in fact this is a manner to measure mass stars, observing how much light is redded.

4. In binary systems, it is possible to observe how the system loses kinematical energy, through decreasing its period; in other words, the two rotating objects are getting closer, until eventually they will collide with each other. This effect has been observed since the 70's in the pulsar binary PRS 1913, explaining it as the releasing of gravitational radiation is the accomplishment that deserved the Nobel prize to Russel and Hulse in 1993.

How are these effects predicted by Einstein's equations?

Equations (2) are solved assuming the simplest geometry: spherical symmetry, i.e we suppose that the metric tensor depends only in one radial coordinate r, which measures the distance to a center. To generate this geometry, a massive body should stay at that center. We also assume a stationary system, i.e. the metric does not depend on time; a spacetime with these features has the form,

$$ds^2 = -f(r)dt^2 + [f(r)]^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{4}$$

where $f(r)$ is determined from Einstein's equations, which in this case reduce to just one equation:

$$\frac{d}{dr}(rf(r)) = 1, \tag{5}$$

whose solution is $f = 1 + C/r$, where C is a constant that can be determined using the condition that at large distances from the center ($r \to \infty$), the gravitational field is very weak, tending to vanish. This requirement lead us to determine the constant as the mass M of a body located at the origin $r = 0$. The line element of this spacetime is,

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{6}$$

This metric is known as Schwarzschild solution and was found by Karl Schwarzschild in 1916. In spite of being the simplest possible solution of Einstein's equations, it has been of huge usefulness to derive some effects or predictions as well as to lead to the prediction of the existence of black holes, very compact massive objects.

Assuming that the sun with mass $M_\odot$ is located in the center of our reference system, our solar system, and assuming that planets, having much smaller masses, do not disturb Sun's gravity, then the planets move along the minimal curves or geodesics, i.e. the planetary orbits are the geodesics in the spacetime produced by the mass of the Sun.

Then we can compare the predicted orbits with the real ones. Neglecting the mass of planets, as gross as it may seem at first sight, is actually a good approximation: just think that Sun's mass (10^{30} kg) is one thousand bigger than Jupiter's, the most massive planet, whose mass is of about 10^{27} kg; therefore the mass ratio is $M_{\mathrm{Jup}}/M_{\odot} = 10^{-3} = 0.001$, quite small. By studying the geodesics of the resulting system, one finds the precession of Mercury's perihelium. In the next section we will introduce the concept of a black hole and comment about the observational astrophysical evidence that points to the real existence of such amazing object.

BLACK HOLE: THE DARK OBJECT

The concept of a black hole (still not so called at the time) was born in 1783 in the mind of an Englishman, John Michel. He reasoned as follows: Knowing the Newtonian gravity law, since every planet attracts bodies, then it could happen that a planet exists with such a great gravitational force, that even light could not escape from it. Implicit is the idea that light is composed by small corpuscles traveling in straight line; such a model for light was accepted at the time, the *Newtonian corpuscular light theory.* The same idea appears in the treatise "Exposition du système du monde" (The System of the World) of Pierre Simon Laplace in 1793. Later on, the wave theory of light proposed by Huygens explaining difraction, reflection, and other wavy effects of light, substituted the newtonian corpuscular theory and the concept of the *dark object* was forgotten.

One century later, the dark objects were brought back to stage. About 1928, in the astronomy circles, the mistery of a strange kind of stars, called white dwarfs, was debated. These stars, according to the observations, were incredibly dense. From its observed luminosity and orbit, the density of one of these stars, Sirius B, turned out to be about $61000\mathrm{gr}/\mathrm{cm}^3$, which is huge compared with the density of the denser materials known in Earth, for example metals: The density of steel is $7.85\mathrm{gr}/\mathrm{cm}^3$, that of lead is $11.3\mathrm{gr}/\mathrm{cm}^3$, and that of platinum is $21.46\mathrm{gr}/\mathrm{cm}^3$. The mean density of Earth is $5.5\mathrm{gr}/\mathrm{cm}^3$ while that of the Sun is about $1.4\mathrm{gr}/\mathrm{cm}^3$. This is why the enormous density of Sirius B was a complete mistery, no one could imagine what kind of material was it made of.

Observations of the redshift of light coming from the star confirmed its huge density: according to einsteinian gravity, light coming from that star should have a redshift thirty times larger than light coming from the Sun. This was verified by W. S. Adams at Mont Wilson Observatory in Pasadena, Cal.

To solve the mistery it was necessary to incorporate the effects of quantum mechanics, the theory of the microscopic world, a recently born theory (1926). Subrahmanyan Chandrasekhar, just graduated as a physicist, anxious to contribute to frontier physics, addressed the problem by studying the way in which equillibrium may arise between forces in stars; it occurs when two pressures are balanced: the internal pressure provoqued by nuclear reactions, pushing outside the star, and the gravitational weight, which pushes in the opposite direction, to the center of the star.

As far as 1925, the equilibrium of a star was explained by the internal pressure originated from chemical reactions, producing heat, counteracting against gravity. However, at temperatures as high as the ones tipical in a star, not only fuel is being consumed, but nuclear reactions take place, transforming elements in heavier ones. Moreover, free electrons are wandering very fast, at high speeds (relativistic speeds), in a state of matter called *plasma* or *degenerate electrons*, where atoms have lost their identities as atoms of such and such elements. At energies that high, quantum effects display: electrons acquire wavy features (another character of electrons, its duality wave-particle). The consequence of this behavior is that matter can be compressed to densities higher than the usual ones.

The young Chandrasekhar was updated on the new quantum physics and applied his recently acquired knowledge to solve the mistery of white dwarfs [4]. Working in the detailed balance between the opposite forces acting in stars, he realized that in the state of degenerate electron plasma, the electronic inertia was enhanced as if electrons were heavier. The resistance of such particles to be compressed is different than it is in normal conditions. Let us suppose that a star is compressed, by effect of gravity, augmenting in 1 % its density. How much does the resistance to be compressed change? The answer at that time was that it should increase on 5/3 (166.67%). However, Chandrasekhar, applying his quantum knowledge, ascertained that the wavy nature of electrons drops that resistance to 4/3, thus explaining why the observed stars are so dense. Mistery solved, objects so dense as Sirius B are produced by nature. Nowadays, the measured actual density of Sirius B is 4×10^6gr/cm^3.

Even more, if the star is massive enough, then the pressure outwards can not compensate the gravitational pressure inwards, thus making the external mass layers collapse inwards, forming a denser object. The critical mass for this to occur is the so called Chandrasekhar limit equal to 1.4 $M_\odot$. Collapse is followed by an explosion that ejects much of the material of the star (a supernova explosion), and after that the remaining core can be a white dwarf, a neutron star or a black hole, depending on the remnant mass.

Therefore, theoretically, the existence of objects so dense that even light is trapped inside them is not forbidden; so they are black undetectable objects. But if we can not see them, if even light cannot escape from them, how can we be sure of their existence? The answer is that matter surrounding these objects is pulled into them. When a black hole is close to a regular bright star, the external layers of the latter are pulled towards the former and fall into it. This matter, mostly gaseous matter, as falling to the black hole, is heated up and starts emitting light in a wide spectrum of energies, particularly high energy beams or X-rays. This radiation could be detected when satellites were launch (1971) with Geiger counters inside; then several X-ray souces were detected, and it was suspected that the emitter object was close to a compact object (neutron star, black hole). The described mechanism to detect black holes was suggested in the mid sixties by Zeĺdovich and Novikov, two Russian physicists.

All sized black holes

About 1974, a particular X-ray source was detected, and when looking for its invisible companion, a pulsar was detected by its radio waves. The two objects forming a binary system (star-black hole, star-pulsar, etc.) cannot be detected by the same apparatus, since they release radiation in different frequencies. That radiaton may be X-rays (gases falling into a compact object), radio waves (pulsar, neutron stars) or visible light (white dwarf); and to detect each type of radiation requires a different apparatus.

Nowadays many of these objects have been observed, and there are many candidates to black holes; so that we firmly believe now that they can exist in nature [5]. Moreover they are present in various sizes: small black holes with masses from 1.4-20 $M_\odot$; giant black holes living in the centers of galaxies, with masses of billions $M_\odot$; and medium size black holes with masses of about 5000 $M_\odot$, that have been observed in globular clusters.

Small black holes are probably located in binary systems, i.e. with another compact object as companion. Several aspirants are observed in this category. This objects are looked for once a X-ray source has been detected, X-ray radiation emitted by the gases of the external layers of companion while falling into the hole. X-ray radiation is detected by satellites outside the Earth atmosphere, which (fortunately!) protects us from receiving that harmful radiation.

At the far end of the electromagnetic spectrum, the very low energetic waves are radio waves, whose wavelenghts can be of meters. They can be observed on Earth with the aid of huge parabolic antennas. Penzias and Wilson in 1965 detected this kind of radiation coming from outside the Earth. Later on it was akcnowledged that this radiation is like a thermal bath surrounding the whole universe with an almost uniform temperature of 2.7 $^\circ$K, which is called the *cosmic microwave background*, that is thought to be the remnant of the Big Bang.

This kind of radiation was observed by Karl Jansky, since 1939, coming from the cosmos, more precisely, from the center of our galaxy. Nobody paid attention to this discovery, but few years later more observations of this kind appeared. This radiation comes from giant black holes situated in the center of almost all galaxies. It is thought that these are black holes billions of times more massive than stellar black holes. In this respect the discovery of giant black holes was unexpected. After the World War II, new tecnology developed during the war (radars to detect the enemy) began to be applied to observations in the cosmos. Radars are emisors of waves that are reflected in some object; the reflected waves allow to determine the location of the reflector. Scientists in England and Australia started to be interested in that kind of observations, to detect radiowaves coming from outer space. To determine the location of the radiowave sources is hard, because large plate antenas are needed, due to the large wavelenght. Alternatively, several antennas can be arranged to cover big areas of several square kilometers, and by trianguling signals in this way, it is possible to determine the location of the source. Signals come from far away galaxies. Some of these sources also emit visible light that make scientists think that they are stars or quasi-stars, quasars.

Up to recently, black holes were supposed to come only in the two sizes that we just mentioned. However, there is something intermediate, some kind of medium-sized black hole. About 2002 two objects, that look just as black holes should look, were detected. The first of them is in clusters M15, with 4,000 $M_\odot$; the other one in G1 with 20,000 $M_\odot$. They are not alone, nor are they in the interior of galaxies, but in globular clusters, which are assembles of many stars.

Stars join in distict types of conglomerates. Globular clusters consist of the oldest stars. According to the Space Telescope Spatial Hubble team, if these clusters have black holes, they are probably there from the very beginning of the formation of stars in the universe. On the other hand, globular clusters are quiet places, nothing to do with the violent centers of galaxies with those giant black holes in there. Medium-sized black holes, such as those just found by Hubble, could be the seeds of future supermassive black holes, as time goes by. The measurements of masses by Hubble are based on the velocity the stars rotate with, around the dense centers of globular clusters. This method yields direct measurements of the masses of black holes.

Black holes: the theory

The old concept of a black hole emerged again in the context of general relativity, as one of the solutions of Einstein's equations. Recall Schwarzschild solution,

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \qquad (7)$$

Notice that if $r = 2M$, then $(1 - \frac{2M}{r}) = 1 - 1 = 0$, this means that one direction, the t direction vanishes, while the r-direction becomes infinite, since $1/0 \to \infty$. In other words, there is some trouble on the locus of the sphere of radius $r = 2M$. Nevertheless, for the celestial objects known so far, one never reaches such a distance (which is called the Schwarschild radius), since it is really small. In the geometrized unit system, where the gravitational constant $G = 1$ and the speed of light is $c = 1$, measuring the mass in lenght units, the Schwarzschild radius of the Sun is $r = 2M_\odot = 2.94$km, while that of Earth is $r = 2M_T = 0.88$ cm, so for astronomical objects of regular density, their Schwarzschild radius lies inside the object and there is no way any observer can reach it.

The horizon

The existence of the locus $r_G = 2M$, called the horizon, sets out several interesting conceptual questions. Let us consider an object so hugely dense and so small that its Schwarzschild radius is located outside it; and let us suppose that we can reach the place where $r = 2M$. What happens when we pass through that place? Notice in the line element from Eq. (7) that when we cross $r = 2M$, passing from $r > 2M$ to $r < 2M$ the factor $(1 - \frac{2M}{r})$ changes its sign from positive to negative. This is, the line element will change its signature from $(- + ++)$ to $(+ - ++)$. But we were measuring time with the negative coordinate $(-dt^2)$, while the distance to the center was r; the change in sign means that time coordinate now is r, while t is now a spacelike coordinate inside the sphere of radius $r = 2M$. Then inside the horizon, the meaning of our spacetime coordinates has changed, now r measures time and t measures the distance, and spacetime is not static anymore. However, an observer traveling towards the black hole would traverse the horizon without feeling nothing strange. Except that he/she would not be able to go back past the horizon again, and will advance until he/she crashes with $r = 0$, *the singularity*, a place where gravitational forces are so enormous that will destroy any object. Meanwhile, people observing from outside of the horizon will see that periodic signals (if the traveler is sending them) coming from the black hole will diminish their periodicity until they die out.

The singularity

For many years it was thought that at $r_G = 2M$ there existed a singularity, this meaning a point in spacetime where and when curvature is infinite, our equations lose their validity and we do not really know what happens there. This opinion held until 1933, when Lemaitre found that the so-called *Schwarzschild singularity* is not really a physical singularity, but only a bad choice of coordinates, and a better choice shows that nothing strange happens there.

To illustrate it, we will create a coordinate singularity: Let us consider the two-dimensional line element $d\sigma^2 = dx^2 + dy^2$. By means of the coordinate transformation given by $\xi \mapsto \frac{x^3}{3}$, the previous line element becomes $d\sigma^2 = (3\xi)^{-4/3} d\xi^2 + dy^2$; which diverges at $\xi = 0$, thus giving us a singularity which is due to the choice of coordinates only.

It often happens that the bad choice is not as obvious as in the previous example. Therefore to discover if the divergence of the line element or singularity really exists or it is just a consequence of the choice of coordinates, we should calculate quantities that do not depend on the coordinate system. These quantities are the *invariants* (with respect to changes of coordinates). This curvature invariants are scalar quantities that we can calculate from the curvature tensors by contracting their indexes. If the invariants at some points become infinite, then the singularity is real, since those curvature invariants will diverge in any coordinate system. On the contrary, if invariants are finite all over the coordinate range, then infinities in the metric components g_{ij} are an effect of the coordinates, and there does not really exist any singularity. For instance in Schwarzschild solution, the invariant

$$R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} = \frac{48M^2}{r^6}, \tag{8}$$

is finite for any value of r, and in particular there is no problem at $r = 2M$ (we get a (finite) value of $3/4M^4$); however at $r = 0$, we actually get infinity, regardless of the choice of coordinates: this singularity is real and will be noticed by any observer. This quantity is invariant because all of the tensor indices appear contracted, thus no change of coordinates will change the value of that expression.

In 1960, M. D. Kruskal found a coordinate system such that the line element is finite even at $r = 2M$, thus showing the good behaviour of spacetime at that point.

Quantum Mechanics and black holes

So, a black hole is an object that do not allow light to escape. We will never be able to *see* it directly but only indirectly, through the radiating matter that falls into it. Such is the prediction of general relativity: when a star collapses forming a black hole, the final object can be completely described by two numbers: its mass and its angular momentum (how fast it rotates); this claim is known as the *no hair theorem* (although, as K. S. Thorne points out, it should really be known as the *two hair theorem*).

In 1963, the Australian R.P. Kerr derived a solution of Einstein's equations that describes the spacetime created by a rotating body with mass M and angular momentum per mass unit a; the corresponding line element, now known as Kerr metric, in coordinates (r, θ, ϕ, t), is given by

$$\begin{aligned} ds^2 &= \rho^2(\frac{dr^2}{\Delta} + d\theta^2) + (r^2 + a^2)\sin^2\theta d\phi^2 - dt^2 \\ &\quad + \frac{2Mr}{\rho^2}(a\sin^2\theta d\phi - dt)^2, \end{aligned} \tag{9}$$

where $\rho^2(r,\theta) = r^2 + a^2\cos^2\theta, \quad \Delta(r) = r^2 - 2Mr + a^2$. This spacetime has been very useful in determining many properties of black holes that rotate, as most of them should, since they are born from rotating stars. When this solution is perturbed (this meaning that one considers its metric components to be $g_{ij} = 1 + \varepsilon h_{ij}$, where ε is very small), physically it represents the fact that the hole interacts with tiny waves or some other tiny distortions, but only tiny things, and we determine what happens. These probes or *perturbations* get dispersed leaving the hole as it was at the beginning. Therefore, we can interpret that black holes do not care about small changes in their immediate neighbourhood (a black hole is a *stable solution*); moreover they do not depend on the kind of material they originally came to be from. All the final black hole inherits is the total mass M and total angular momentum a. This is, as we said, the *no hair theorem*.

Therefore black holes are affected by gravitational forces only, but the main domain in which gravitational forces make feel their influence is the domain of the macroscopic. In the microscopic world gravity is negligible, since microscopic masses are tiny. In the microscopic domain, other forces are the dominant ones: electromagnetic and strong nuclear force. This is the quantum world. Only in very particular scenarios, will gravity be as important as any other of these forces. For instance in the neighborhood of a black hole, where the gravitational field is huge. In this sense we can think of Einstein's theory as a field approximation that does not describe all aspects of nature (for example, its quantum aspects). A complete description of nature would then be given by a theory that accounts for both gravitational and quantum forces, i.e. a *quantum gravity*.

Currently, there are several proposals for quantum gravity theory, but they still do not predict observations that can be tested. What has been done a lot and with a pretty reasonable success, is to take a part of gravity and a part of quantum theory, in what is called a semiclassic or semiquantum approach; assuming a curved and fixed scenario, where there exist quantum fields that do not affect the curvature of spacetime. For instance, the neighbouring spacetime of a black hole (Schwarzschild solution) interacting with photons or scalar particles that obey quantum equations. In this approach, geometry in the left hand side of Einstein's equations (2) is curved but fixed, while in the right hand side, matter (or the energy-momentum tensor) is the expected value of a quantum field, this is,

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \langle T_{\mu\nu} \rangle. \tag{10}$$

>From this scheme, we can get amazing and unexpected facts: black holes evaporate!

Black holes evaporate!

For us, vacuum is related to the absence of matter. However, according to quantum mechanics, vacuum is not empty space. Even when there is no matter, vacuum may fluctuate: even when the total energy is zero in the mean, it may happen that small regions with positive energy are compensated by other small regions with negative energy; these are *vacuum fluctuations*. We do not know if this is really the way nature is, but it is a model we have. Since the model works, i.e. it predicts observable effects, then the model is good. One of the predictions is that vacuum fluctuations cause that some atoms emit radiation spontaneously. Another predicted and observed effect is the Casimir effect, which consists in two paralell metal plates, in a vacuum enviroment, which attract each other without any apparent cause (i.e. in absence of any external force). One of the first good predictions of this vacuum fluctuations model was related to the hydrogen spectrum or energy levels, in some fine lines called hyperfine structure. Therefore, vacuum (according to quantum mechanics) is not a bored empty state, but an interesting and dynamical one that awakes when interacting with electric or magnetic fields, creating and destroying elementary particles all the time.

Now let us go back to the gravitational fields, modelled as curvature or wrinkles in spacetime. May they present similar effects? How is vacuum for gravitational field? The logic implication is that when vacuous spacetime interacts with other fields, a very similar creation and destruction of elementary particles takes place, in a way analogous to that of electromagnetic vacuum in which electrons and positrons are created and destroyed. If gravitational energy fluctuations do exist, spacetime suffers very tiny fluctuations in curvature, so that there is no net creation of particles and energy remains the same.

By the year 1974, Steven Hawking found that near the horizon of a black hole, particles can be created if an external field is turned on. In a state of (initially) zero energy of the probing field, a pair of particles, one with positive and the other with negative energy, might be created. It may happen that the particle with negative energy falls into the black hole, while the one with positive energy escapes, thus really coming into existence. Now if this happens a lot, it would appear as if the black hole radiates particles, or some kind of radiation, with an energy associated. We could say then that a black hole has a temperature! It turns out that this temperature is inversely proportional to the mass of the black hole. For a black hole of one solar mass, this temperature is too small to be detected. The tinier the black hole is, the greater its temperature will be. For instance, a black hole with mass 10^{18} Kg (mass in the range of that of a mountain) would have a temperature of 5000°K, and its aspect would be like that of a white bulb of one miliwatt.

In radiating, the black hole loses mass becoming smaller and smaller, and so it increases its temperature and eventually, having loss all its mass, it will vanish completely;

then we could say (maybe abusing of languaje) that the black hole had evaporated. A black hole of a few kilograms (something really hard to create) would evaporate in a milisecond, releasing more energy than an atomic bomb.

This kind of speculations gave rise to questions along the lines of whether there is something like a thermodynamical entropy associated to these processes?. And, if this is so, which are the microscopic states responsible for this entropy?, or how can we derive such a property from a microscopic statistical model?. And also, what happens with all the information carried by the matter that is swallowed by the black hole?

Black hole entropy

Recall that entropy is a way of measuring the extent to which a system is "disordered"; to derive entropy from first principles, a microscopic model should be assumed. Recall also that the second law of thermodynamics says that the entropy of a system never decreases. Well, it turns out that for black holes there is a rule, very analogous to the entropy law, provided one identifies entropy and *horizon area*, the latter being given by $A = 4\pi r_h^2$, where r_h is the horizon radius.

Hawking and Bekenstein arrived to the conclusion that, when a black hole swallow things, its horizon area increases, and there is no process which make this area decrease. Then horizon area and the entropy are two of a kind. However, if one wants to go farther, then we should be able to figure out a microscopic model that explains the black hole *entropy*. Some calculations give the huge number of $S \approx 10^{79}$, which still can not be explained. This problem, still unsolved, is very complicated and the answer would have to involve a correct theory that joins gravity and quantum mechanics, and the formulation of a statistical model in terms of such a theory.

GRAVITATIONAL WAVES

Gravitational waves are another amazing feature that comes out of general relativity; more than that, gravitational waves have been *indirectly* observed. Binary pulsars show several relativistic effects; observations in their diminishing rotational periods constitute the most recent confirmation of the existence of gravitational radiation.

In 1974, a pulsating source was discovered: it was the PSR 1913+16 pulsar. Shortly afterward, this pulsar was found out to be actually two compact bodies in rotation one around the other. This system has been observed for 37 years and by now all of its features are well known: the orbits, the frequencies of the pulses; the masses of the two bodies (which are 1.4411 $M_\odot$ and 1.3873 $M_\odot$); and the duration of the orbital period: 7 hours and 45 minutes. We know that one of the objects is a pulsar, since we get periodic signals (radiowaves) from it; while the other must be a neutron star since it doesn't eclipse the pulses. The orbit of the system has a size of about the solar radius, 695 000 km.

Two objects rotating around each other will generate gravitational waves. The two objects feel mutual attraction and keep getting closer and closer (in astronomical times) until they eventually collide. During all the process, gravitational waves are released, and therefore the system keeps losing kinetic energy. Since the objects are getting closer, the orbit gets smaller and consequently, their period of revolution becomes shorter as time elapses.

The aforementioned binary pulsar is some 16 thousand light years from us. Altough the signals are weak, after four years of observations the diminishment of the period was noticed: the orbit was shrinking. This means that the system is losing energy and the two stars are falling toward each other. If it is assumed that the released energy is gravitational radiation, and the theoretical calculation of the reduction of the period is plotted jointly with the observed data, the concordance between both is astonishing [6].

Every year the orbital period decreases by 75×10^{-6} of a second. With each revolution of one star around the other, they get closer by one milimeter. So they will collide in 240 million years from now. Other relativistic effects are also detected in this binary system: the precession of the orbit is vigorous, as big as 4.2 degrees per year, 35 thousand times greater than that of Mercury's orbit.

Nowadays, about 50 of those systems are being observed, and some of them remarkably exhibit several relativistic effects. For instance, the two pulsars binary system PSR JO737-3039 A/B, whose orbit shrinks (even faster than PSR 1913+16) 7 milimeters every day, so that the two pulsars will collide in 85 million years. Moreover, since both bodies are pulsars, they emit radio pulses with a measurable frequency, and it is observed how one signal eclipses the other periodically.

These observations are an indirect proof of the existence of gravitational waves predicted by general relativity. In 1993 Rusell Hulse and Joseph Taylor were awarded the Nobel Prize in Physics by their discovery of the PRS 1913+16 pulsar, as an acknowledgement from the scientific community to the reality of gravitational radiation. As a sequel, several groups all around the world began to think how to detect *directly* the gravitational waves.

Some theory on gravitational waves

As was mentioned, gravitational waves are distortions of the spacetime curvature; perturbations that transport energy. Theoretically, the existence of gravitational waves was found not long time after the introduction of general relativity. Let us consider the spacetime, characterized by the metric tensor, plus a small tiny perturbation,

$$g_{\mu\nu} = \eta_{\mu\nu} + \varepsilon h_{\mu\nu}, \tag{11}$$

where $\eta_{\mu\nu}$ is the Minkowski metric tensor, $\varepsilon \ll 1$ is a dimensionless small parameter and $h_{\mu\nu}$ is the gravitational perturbation. Once we know the metric tensor the curvature quantities can be calculated: the Christoffel symbols $\Gamma^{\gamma}_{\mu\nu}$, the Ricci tensor $R_{\mu\nu}$, the curvature scalar R. All these quantities are then plugged into Einstein's equations, and, throwing out all terms of order greater than one in ε, we get the linearized Einstein's equations,

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) h_{\mu\nu} = 0, \tag{12}$$

which is the wave equation, with velocity of propagation c! This tells us that gravity may have a wavy existence, at least theoretically; and moreover, this waves travel with the velocity of light. Einstein himself found this result by 1916, the year after the publication of the general relativity theory. He was skeptical about the real existence of the gravitational field in a wavy form; he thought it was a result of the linearization process. Other physicists had doubts about gravitational waves as well. In the fifties there was still a debate on whether waves that appear in a particular reference frame, may disappear in another. Theoretically, it was found that this kind of waves would carry energy, but energy turns out not to be a well established concept in general relativity.

However, after 1993, the idea of the real existence of gravitational waves began to permeate among the scientific community. How can we know that a gravitational wave is passing through?. If test particles are arranged forming a ring, located transversal to the wave, the ring will stretch and shrinks successively so that it takes the form of an ellipse, then a circle, then again an ellipse, etc; and the same but with the ellipse rotated by an angle of $\pi/4$ if the wave carries another polarization. The spacetime stretches and shrinks transversally to the direction of propagation of the wave.

The theoretical approach, starting from the linearized gravity, is very similar to the study of the electromagnetic field, and there are in fact many common features: both waves are transversal to the propagation direction; and the wave equation for gravity, when in a special reference frame (tt-gauge), has only two degrees of freedom or polarizations: “$+$" and “$\times$", just like electromagnetic waves.

However, there are important differences between electromagnetic and gravitational waves, mainly in the way they are generated and how they interact with the medium they traverse: electromagnetic radiation can be generated by a time varying electric dipole. In contrast, a pulsating spherical body won’t generate gravitational radiation: the gravitational dipole does not exist, because the gravitational mass (which in this context works as the “gravitational charge”) and the inertial mass are equal, and so the conservation of momentum would forbid dipolar gravitational radiation. Thus, the first radiative term in a multipole expansion of gravitational radiation is the quadrupolar one. Calculating the radiated power by a binary system with masses m_1 and m_2, rotating in an elliptic orbit of excentricity ε and larger semiaxis a we obtain [7]

$$P = -\frac{dE}{dt} = -\frac{8}{15}\frac{G^4}{a^5c^5}m_1^2m_2^2(m_1+m_2)F(\varepsilon), \tag{13}$$

where $F(\varepsilon)$ is a function depending on the excentricity, $G = 6.67\times 10^{-11}\mathrm{m}^3/\mathrm{kg\ s}^2$ is the gravitational constant of Newton, and $c = 3\times 10^8\mathrm{m/s}$. Notice that the factor $G^4/c^5 \sim 10^{-52}$J/s is very small, so that in order to generate a measurable gravitational radiation, huge masses are needed, preferably close to each other, rotating at large angular velocities. Let us consider for instance the system Earth-Sun, with masses $m_{\mathrm{Earth}} = 2\times 10^{24}$kg and $m_\odot = 2\times 10^{30}$kg, separated by $d = 1.5\times 10^{11}$m; then the radiated power is $P = 22$ Watts. i.e. the released gravitational radiation is 22 Joules per second. As a consequence, the orbit Earth-Sun is shrinking at a rate of 10^{-16}m per day. Hilariously slow! But consider now a binary pulsar, with masses of about one solar mass each, separated by a distance $d = 189\times 10^6$m, then the radiated power is about $P = 4.32\times 10^{26}$ Watts, which is quite huge.

Direct detection of gravitational waves

Back in history, the first attempts to directly detect gravitational waves took place in the decade of 1960, by Joseph Weber in Maryland. Weber figured out how to detect waves by using the resonance phenomena, which occurs when the frequency of a wave interacting with a system is the same than the characteristic frequency of the system. If this happens, the system releases a lot of energy. So Weber designed an apparatus consisting of a huge aluminium cylinder (3.5 Tons) covered with small pieces of piezoelectric material; this material emits electric signals when distorted by some forces, producing an electric current. So, the idea was that the gravitational wave, while passing through the cylinder, and if its frequency was the same as the resonant characteristic frequency of the cylinder ($\sim$ 1000Hz), would generate a resonance which would in turn distort the cylinder and an electric current would be detected. The tight range of frequencies in the apparatus was one of the problems; however, Weber reported two simultaneously observed signals, one in the facility located in Maryland and the other just a few kilometers away, in Argonne (reported in Physical Review Letters [8]). But no one could ever detect another signal again, and therefore many people was skeptic about Weber's finding and that trend did not prosper ahead.

After the breakthrough of the discovery of the double pulsar PSR 1913+16, and the subsequent certainty that gravitational radiation was being observed, several groups around the world, with renewed efforts, began to built facilities aimed to detect directly gravitational waves.

The way in which people are now trying to detect gravitational waves, is by means of interferometers. The interferometer is an apparatus that measures the distance traveled by light in two perpendicular directions. A ray of light is separated into two rays and using a mirror, one half is sent in one direction and the other half in the perpendicular direction; at a certain distance, mirrors reflect each ray, so they go back to the initial point. In this way, if space stretches or shrinks in some direction, each half ray will traverse different distances and we will know it by observing the difference in the phases of both rays.

This kind of apparatus was used at the end of the nineteenth century to prove the nonexistence of aether. The same idea is now reloaded to detect gravitational waves, but now the mirrors are placed in a massive body. The gravitational wave in passing will stretch and shrink spacetime, and the perpendicular paths will change their lenght. If this happens, the traversing rays of light will return to the initial point in a different phase (the wavefronts would not be in sincrony as when the ray left the source). Roughly speaking this is the idea to design gravitational wave detectors. Of course, there are many technical issues to solve. One of them is noise: the expected signal is so tiny that the seismic movements of Earth, or ocean tides caused by the gravitational force of the Moon, can mask or hide it. There are also external magnetic fields and cosmic rays. In order to address this difficulty, the arms of the interferometer are kept in vacuum. Another one is size: an interferometer trying to detect gravitational waves should be really big, since the expected frequencies are very tiny. So, the mirrors are placed in such a way that the rays of light go back to the initial point after being reflected about 100 times.

A theoretical problem is to determine the expected range of frequencies to be detected.

And the expected frequency depends on the phenomena that produces the gravitational waves. The range of frequencies is wide, from 10^{-15} Hertz from the Big Bang, through 10^{-14} Hertz coming from supermassive black holes in the center of our galaxy or from binary systems far away, to 10^2 Hertz from binary systems not so far from us.

Some facilities are LIGO, VIRGO, AMANDA. LIGO has made runs but no signals have reached the laboratory; however, the frequency of events that release gravitational waves is not so high as to be in panic for not detecting signals yet. In the next two years, an advanced stage of the facilities will be constructed in order to reach more sensitivity. This will augment the number of detectable phenomena.

Lastly, I want to comment on a modern branch of research that has turned out to be a very fruitful one, namely *numerical relativity* [9]. It deals with the numerical simulation of events such as the colision of two black holes, or how do gravitational waves interact with a black hole horizon. Many solutions of Einstein's equations are stationary, i.e. they do not depend on time; therefore, to figure out the evolution of some of them, Einstein's Equations are set as numerical equations that are made to evolve, chosing a particular initial time which is then let to vary and see what happens. There are several issues to take care of: for one thing, the evolving solutions must at every time be a solution of Einstein equations; this is not an easy task, but in this way both the collision of black holes and the gravitational radiation emitted have been tracked, as well as the process of colapsing of a mass and the subsequent creation of a horizon. Also, the evolution of some binary systems has been modeled. From these studies, we can determine the possible frequency ranges in which gravitational waves can be searched for, among other interesting results.

ACKNOWLEDGMENTS

The author acknowledges the Organizing Committee of the VIII DGFM Workshop (Tuxtla Gutiérrez, Chiapas) for the invitation to lecture. Thanks are due to D. J. Fernández Bretón for reading the manuscript and for useful comments.

REFERENCES

1. A. Einstein, *La Relatividad,* Colección DINA, Ed. Grijalbo (1970).
2. G. Gamow, *El breviario del Señor Tompkins,* Fondo de Cultura Económica (1985).
3. B. Mielnik, *Ficción científica y relatividad general*. Avance y Perspectiva, **35**, 34-40 (1988). (grupogr.fis.cinvestav.mx/fcrg.html)
4. K. S. Thorne, *From black holes to time warps: Einstein's outrageous legacy* W. W. Norton & Co. (1994). Or its Spanish version: *Agujeros negros y tiempo Curvo*, Ed. Crítica, Barcelona (1995).
5. R. M. Wald, Editor, *Black Holes and Relativistic Stars*, The Univ. Chicago Press (1998)
6. J. H. Taylor and J. M. Weisberg, *A new test of general relativity:Gravitational radiation and the binary pulsar PSR 1913+16*, Astrophysical J. **253**908 (1982).
7. P. C. Peters and J. Mathews, *Gravitational Radiation from point masses in a Keplerian Orbit*, Phys. Rev. **131** 435 (1963).
8. J. Weber, *Gravitational-wave-detector events,* Phys. Rev. Lett. **20** 1307(1968).
9. M. Alcubierre, *Introduction to 3+1 Numerical Relativity*, Oxford Univ. Press (2008)